SS-ZG548: ADVANCED DATA MINING

02 Incremental Data Mining

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Recap: Data Mining

Data mining `does KDD: knowledge discovery in databases`

KDD involves five components 1) collection of data, 2) preprocessing, 3) transformation, 4) data mining, and 5) interpretation.

- Data mining has three parts **Model**, **Preference**, and **Search**
- Two broad categories 1) **Predictive** if we focus on new data involving classification, regression, time series analysis, and prediction 2) **Descriptive** when we want to understand/describe the data itself involving clustering, summarization, association rules, or sequence discovery

Issues

Human interaction, Overfitting, Outliers, Large dataset, High dimension, Multimedia data, missing data, irrelevant data, noisy data, changing data.

Association Rules

Association rules tries to do linked analysis.

Example: Whether same products are selling together?

- $I = \{i_1, i_2, i_3, ...i_m\}$, $T = \{t_1, t_2, t_3, ..., t_n\}$ and $t_i \subseteq I$

[1] Mining association rules between sets of items in large databases, *R Agrawal, T Imielinski, and A Swami*, SIGMOD, 22(2), pp 207–216, ACM-1993

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- Can you see: Subset of frequent items is also frequent

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- Apriori analysis [1]

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- Minimum support count should be maintained
- Can you see: Subset of frequent items is also frequent
- Apriori analysis [1]

Let's do it:

$t_1 = (1, 3, 4)$, $t_2 = (2, 3, 5)$, $t_3 = (1, 2, 3, 5)$, $t_4 = (2, 5)$, $t_5 = (1, 3, 5)$ and minimum support count be 2

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Association Rules

$t_1 = (1, 3, 4)$, $t_2 = (2, 3, 5)$, $t_3 = (1, 2, 3, 5)$, $t_4 = (2, 5)$, $t_5 = (1, 3, 5)$

Association Rules

$t_1 = (1, 3, 4)$, $t_2 = (2, 3, 5)$, $t_3 = (1, 2, 3, 5)$, $t_4 = (2, 5)$, $t_5 = (1, 3, 5)$

Symb	Sup
$\{1\}$	3 ✓
$\{2\}$	3 ✓
$\{3\}$	4 ✓
$\{4\}$	1
$\{5\}$	4 ✓

Association Rules

$t_1 = (1, 3, 4)$, $t_2 = (2, 3, 5)$, $t_3 = (1, 2, 3, 5)$, $t_4 = (2, 5)$, $t_5 = (1, 3, 5)$

Symb	Sup
{1}	3 ✓
{2}	3 ✓
{3}	4 ✓
{4}	1
{5}	4 ✓

Symb	Sup
{1,2}	1
{1,3}	3 ✓
{1,5}	2 ✓
{2,3}	2 ✓
{2,5}	3 ✓
{3,5}	3 ✓

Association Rules

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Symb	Sup
{1}	3 ✓
{2}	3 ✓
{3}	4 ✓
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{5}	4 ✓

Symb	Sup
{1,2}	1
{1,3}	3 ✓
{1,5}	2 ✓
{2,3}	2 ✓
{2,5}	3 ✓
{3,5}	3 ✓

Symb	Sup
{1,2,3}	1
{1,2,5}	1
{1,3,5}	2 ✓
{2,3,5}	2 ✓

Association Rules

$t_1 = (1, 3, 4)$, $t_2 = (2, 3, 5)$, $t_3 = (1, 2, 3, 5)$, $t_4 = (2, 5)$, $t_5 = (1, 3, 5)$

Symb	Sup
{1}	3 ✓
{2}	3 ✓
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Symb	Sup
{1,2}	1
{1,3}	3 ✓
{1,5}	2 ✓
{2,3}	2 ✓
{2,5}	3 ✓
{3,5}	3 ✓

Symb	Sup
{1,2,3}	1
{1,2,5}	1
{1,3,5}	2 ✓
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Symb	Sup
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Association Rule Mining

Mathematical model of Association Rule Mining

- Let $I = \{i_1, i_2, ..., i_m\}$ be set of items

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An association rule is an implication
of the form $X \Rightarrow Y$, where $X \subseteq I$, $Y \subseteq I$ and $X \cap Y = \phi$

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- An association rule $X \Rightarrow Y$ has a **support** s in set T if $s\%$ of the transactions in T contains $X \cup Y$

$$support(X \Rightarrow Y) = P(X \cup Y)$$

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- An association rule $X \Rightarrow Y$ has a **support** s in set T if $s\%$ of the transactions in T contains $X \cup Y$

$$support(X \Rightarrow Y) = P(X \cup Y)$$

- The association rule $X \Rightarrow Y$ holds in the transaction set T with **confidence** c if $c\%$ of the transactions in T that contain X also contain Y

$$confidence(X \Rightarrow Y) = P(Y|X)$$

An Example

Find **support** and **confidence** for $X \Rightarrow Y$ in following database

(Z)
(Z)
(Z)
(X,Y)
(X,Y)
(X,Y)
(X,Z)
(X,Z)
(Z)
(Z)

- **support** $= P(X \cup Y)$

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- **support** $= P(X \cup Y)$

$3/10$

- **confidence** $= P(Y|X)$

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(Z)
(Z)

- **support** $= P(X \cup Y)$

  $3/10$

- **confidence** $= P(Y|X)$

  $3/5$

Association Rule Mining (contd..)

For a given *support* and *confidence* the problem of mining association rules is to find out all the association rules that have confidence and support greater than the corresponding thresholds.

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It is a two-step process

1. *Find all frequent item sets:* $\{X : support(X) \geq S_{min}\}$

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1. *Find all frequent item sets:* $\{X : support(X) \geq S_{min}\}$
2. *Generate association rules from the frequent item set:* For any pair of frequent item set W and X satisfying $X \subset W$, of $support(X)/support(W) \geq C_{min}$, then $X \Rightarrow Y$ is a valid rule where $Y = W - X$.

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- Second part is straight forward
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Prior work includes *Apriori*, *DHP*, *partition based*, *TreeProjection*, FP-Tree, and constraint-based ones.

Overview of Apriori

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Example:

Consider $I = \{A, B, C, D, E, F\}$ and transaction $T = \{ t_1 = (A, B, C), t_2 = (A, F), t_3 = (A, B, C, E), t_4 = (A, B, D, F), t_5 = (C, F), t_6 = (A, B, C), t_7 = (A, B, C, E), t_8 = (C, D, E), t_9 = (B, D, E), \}$ and the minimum support be greater then 3.

Apriori at work

Consider transactions T

T_1=(A,B,C)
T_2=(A,F)
T_3=(A,B,C,E)
T_4=(A,B,D,F)
T_5=(C,F)
T_6=(A,B,C)
T_7=(A,B,C,E)
T_8=(C,D,E)
T_9=(B,D,E)

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T_7=(A,B,C,E)
T_8=(C,D,E)
T_9=(B,D,E)

Scan T $\longrightarrow$ C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

Apriori at work

Consider transactions T

T_1=(A,B,C)
T_2=(A,F)
T_3=(A,B,C,E)
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T_6=(A,B,C)
T_7=(A,B,C,E)
T_8=(C,D,E)
T_9=(B,D,E)

Scan T → C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

→ L_1

Item	Supp
{A}	6
{B}	6
{C}	6
{E}	4

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Consider transactions T

T_1=(A,B,C)
T_2=(A,F)
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T_5=(C,F)
T_6=(A,B,C)
T_7=(A,B,C,E)
T_8=(C,D,E)
T_9=(B,D,E)

C_2

Item
{A,B}
{A,C}
{A,E}
{B,C}
{B,E}
{C,E}

Scan T → C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

→ L_1

Item	Supp
{A}	6
{B}	6
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T_8=(C,D,E)
T_9=(B,D,E)

Scan T → C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

→ L_1

Item	Supp
{A}	6
{B}	6
{C}	6
{E}	4

C_2

Item
{A,B}
{A,C}
{A,E}
{B,C}
{B,E}
{C,E}

Scan T → C_2

Item	Supp
{A,B}	5
{A,C}	4
{A,E}	2
{B,C}	4
{B,E}	3
{C,E}	3

Apriori at work

Consider transactions T

T_1=(A,B,C)
T_2=(A,F)
T_3=(A,B,C,E)
T_4=(A,B,D,F)
T_5=(C,F)
T_6=(A,B,C)
T_7=(A,B,C,E)
T_8=(C,D,E)
T_9=(B,D,E)

Scan T → C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

→ L_1

Item	Supp
{A}	6
{B}	6
{C}	6
{E}	4

C_2

Item
{A,B}
{A,C}
{A,E}
{B,C}
{B,E}
{C,E}

Scan T → C_2

Item	Supp
{A,B}	5
{A,C}	4
{A,E}	2
{B,C}	4
{B,E}	3
{C,E}	3

→ L_2

Item	Supp
{A,B}	5
{A,C}	4
{B,C}	4

Apriori at work

Consider transactions T

T_1=(A,B,C)
T_2=(A,F)
T_3=(A,B,C,E)
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T_6=(A,B,C)
T_7=(A,B,C,E)
T_8=(C,D,E)
T_9=(B,D,E)

Scan T → C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

→ L_1

Item	Supp
{A}	6
{B}	6
{C}	6
{E}	4

C_2

Item
{A,B}
{A,C}
{A,E}
{B,C}
{B,E}
{C,E}

Scan T → C_2

Item	Supp
{A,B}	5
{A,C}	4
{A,E}	2
{B,C}	4
{B,E}	3
{C,E}	3

→ L_2

Item	Supp
{A,B}	5
{A,C}	4
{B,C}	4

C_3

Item
{A,B,C}

Apriori at work

Consider transactions T

T_1=(A,B,C)
T_2=(A,F)
T_3=(A,B,C,E)
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T_6=(A,B,C)
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T_8=(C,D,E)
T_9=(B,D,E)

Scan T → C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

→ L_1

Item	Supp
{A}	6
{B}	6
{C}	6
{E}	4

C_2

Item
{A,B}
{A,C}
{A,E}
{B,C}
{B,E}
{C,E}

Scan T → C_2

Item	Supp
{A,B}	5
{A,C}	4
{A,E}	2
{B,C}	4
{B,E}	3
{C,E}	3

→ L_2

Item	Supp
{A,B}	5
{A,C}	4
{B,C}	4

C_3

Item
{A,B,C}

Scan T → C_3

Item	Supp
{A,B,C}	4

Apriori at work

Consider transactions T

T_1=(A,B,C)
T_2=(A,F)
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T_4=(A,B,D,F)
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T_7=(A,B,C,E)
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Scan T → C_1

Item	Supp
{A}	6
{B}	6
{C}	6
{D}	3
{E}	4
{F}	3

→ L_1

Item	Supp
{A}	6
{B}	6
{C}	6
{E}	4

C_2

Item
{A,B}
{A,C}
{A,E}
{B,C}
{B,E}
{C,E}

Scan T → C_2

Item	Supp
{A,B}	5
{A,C}	4
{A,E}	2
{B,C}	4
{B,E}	3
{C,E}	3

→ L_2

Item	Supp
{A,B}	5
{A,C}	4
{B,C}	4

C_3

Item
{A,B,C}

Scan T → C_3

Item	Supp
{A,B,C}	4

→ L_3

Item	Supp
{A,B,C}	4

Generate Association Rules

If $X \subset W$ & $support(X)/support(W) \geq C_{min}$, then $X \Rightarrow W - X$

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- Our frequent item contains
  - $\{A\}_6$ $\{B\}_6$ $\{C\}_6$ $\{E\}_4$ $\{A,B\}_5$ $\{A,C\}_4$ $\{B,C\}_4$ $\{A,B,C\}_4$

Generate Association Rules

If $X \subset W$ & $support(X)/support(W) \geq C_{min}$, then $X \Rightarrow W - X$

Transactions

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  - $\{A\}_6$ $\{B\}_6$ $\{C\}_6$ $\{E\}_4$ $\{A,B\}_5$ $\{A,C\}_4$ $\{B,C\}_4$ $\{A,B,C\}_4$
- Possibilities are
  $A \Rightarrow B$, $B \Rightarrow A$, $A \Rightarrow C$, $C \Rightarrow A$, , $B \Rightarrow C$, $C \Rightarrow B$,
  $A \Rightarrow \{B, C\}$, $B \Rightarrow \{A, C\}$, $C \Rightarrow \{B, A\}$,
  $\{A, B\} \Rightarrow C$, $\{A, C\} \Rightarrow B$, $\{B, C\} \Rightarrow A$
- Let's take $C_{min} = 1.22$

Generate Association Rules

If $X \subset W$ & $support(X)/support(W) \geq C_{min}$, then $X \Rightarrow W - X$

Transactions

T_1=(A,B,C)
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T_9=(B,D,E)

- Our frequent item contains
  - $\{A\}_6$ $\{B\}_6$ $\{C\}_6$ $\{E\}_4$ $\{A,B\}_5$ $\{A,C\}_4$ $\{B,C\}_4$ $\{A,B,C\}_4$
- Possibilities are
  $A \Rightarrow B$, $B \Rightarrow A$, $A \Rightarrow C$, $C \Rightarrow A$, , $B \Rightarrow C$, $C \Rightarrow B$,
  $A \Rightarrow \{B, C\}$, $B \Rightarrow \{A, C\}$, $C \Rightarrow \{B, A\}$,
  $\{A, B\} \Rightarrow C$, $\{A, C\} \Rightarrow B$, $\{B, C\} \Rightarrow A$
- Let's take $C_{min} = 1.22$
- Association rules that qualifies as valid rule are shown green
  $A \Rightarrow B$, $B \Rightarrow A$, $A \Rightarrow C$, $C \Rightarrow A$, $B \Rightarrow C$, $C \Rightarrow B$, $A \Rightarrow \{B, C\}$,
  $B \Rightarrow \{A, C\}$, $C \Rightarrow \{B, A\}$, $\{A, B\} \Rightarrow C$, $\{A, C\} \Rightarrow B$, $\{B, C\} \Rightarrow A$

Thank You!

Thank you very much for your attention!

Queries ?