IS-ZC444: ARTIFICIAL INTELLIGENCE

Lecture-06: Beyond Classical Search

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  - Integrated circuit design
  - Factory floor layout
  - Job shop scheduling
  - Automatic programming
  - Telecommunication network optimization
  - Vehicle Routing
  - Portfolio management

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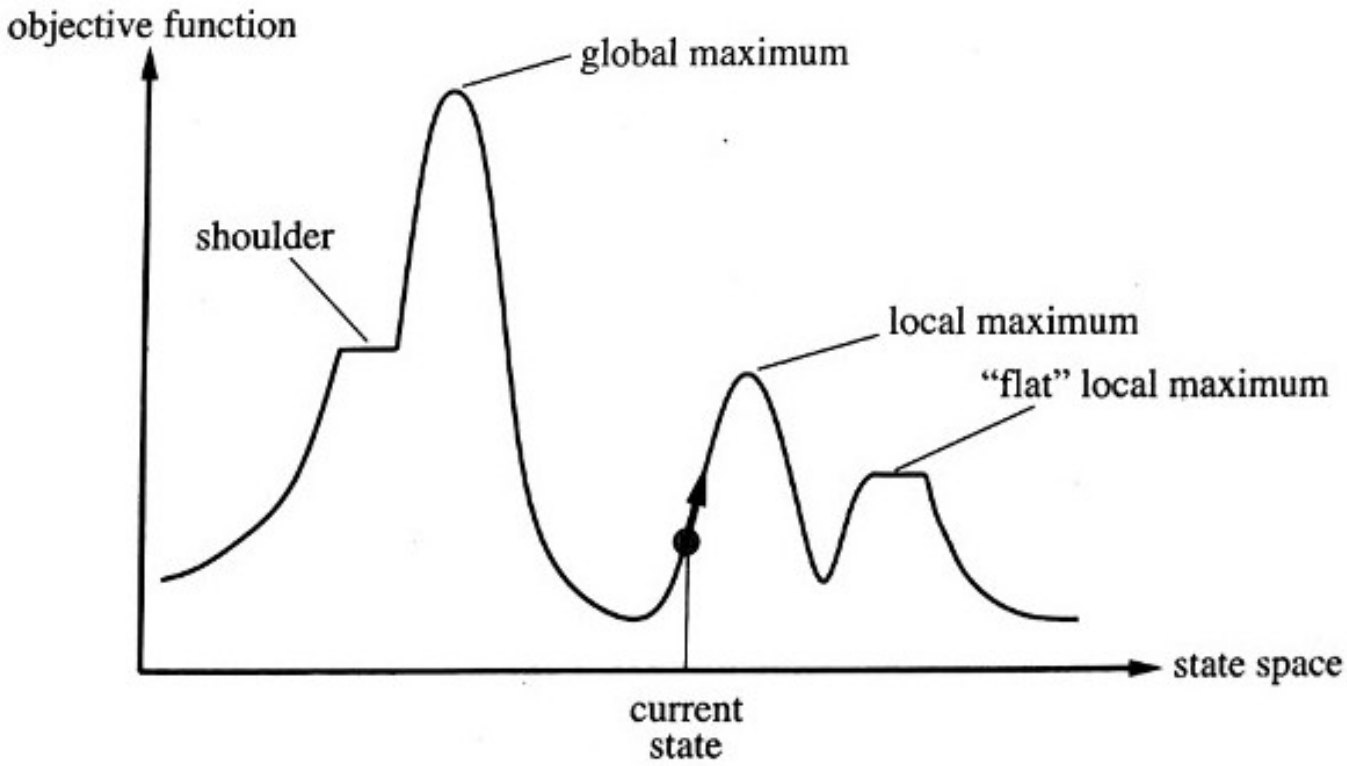
- We go for **local search** algorithms in such scenarios
- These algorithms operate using a single current node (and move to its neighbor). Advantages include
  1. Uses very little memory
  2. Can find reasonable solution in large or infinite (continuous) state space.

Local Search Algorithms

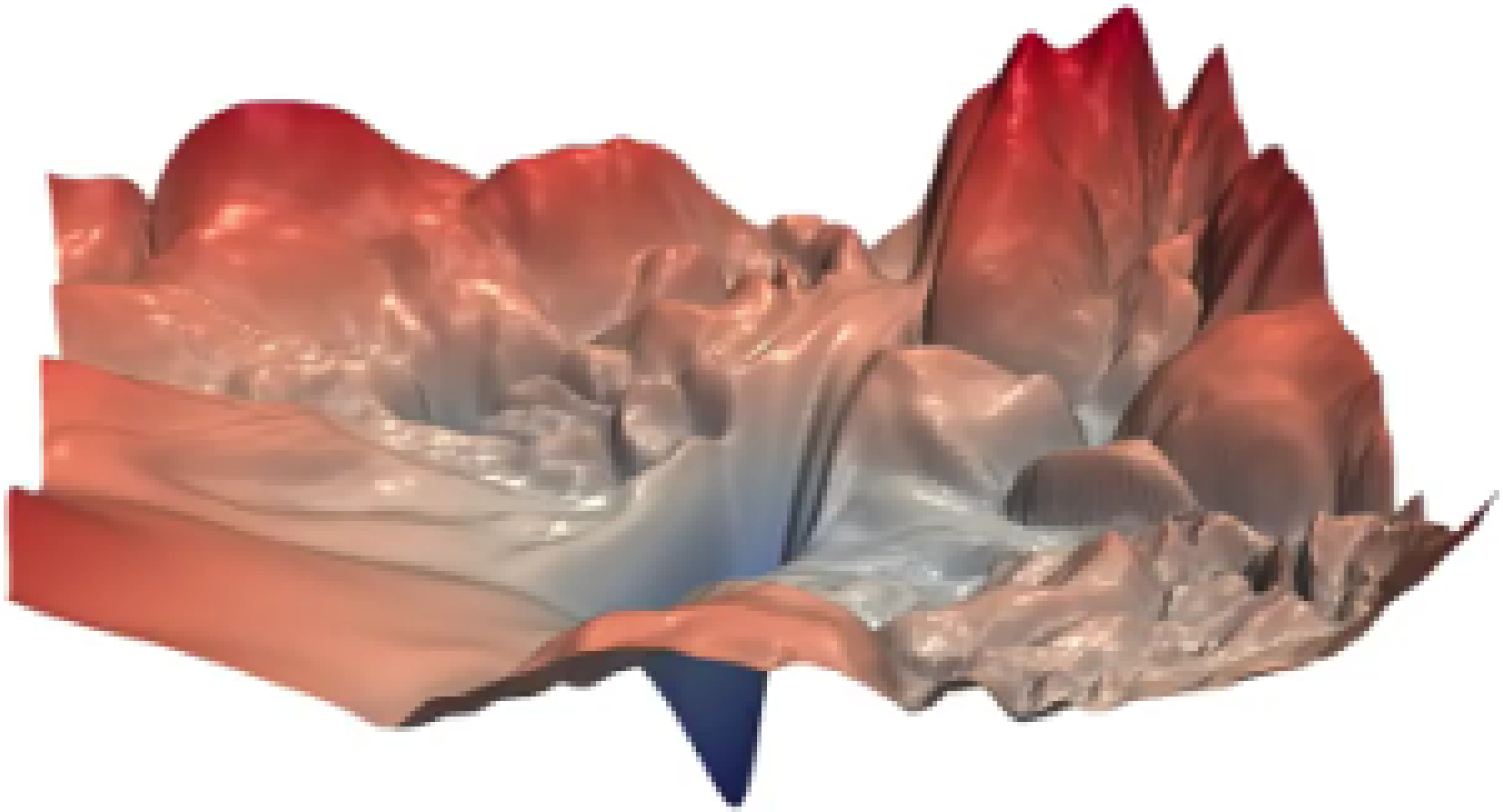
- Local search algorithms can solve pure optimization problems.
- They find optimum state based on some **objective function** [1]

State space landscape can be used to proceed.



[1] Many optimization problems do not fit in standard search model. For example Darwinian evolution uses an objective function **reproductive fitness** but there is no goal test and path cost for this.

Real State Space Landscape



Hill Climbing Search

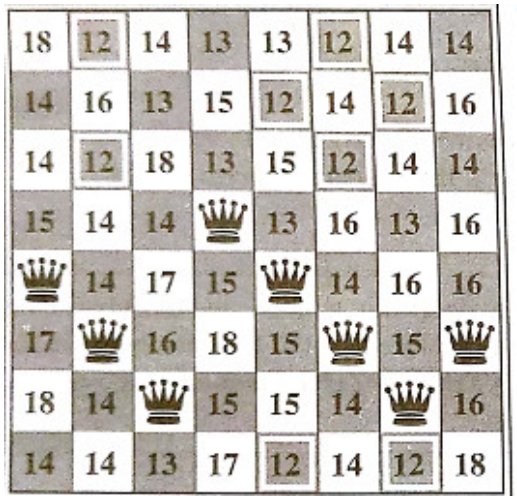
- Steepest ascent version
- Look around and choose the best
- Greedy-local search
- Makes quite rapid progress

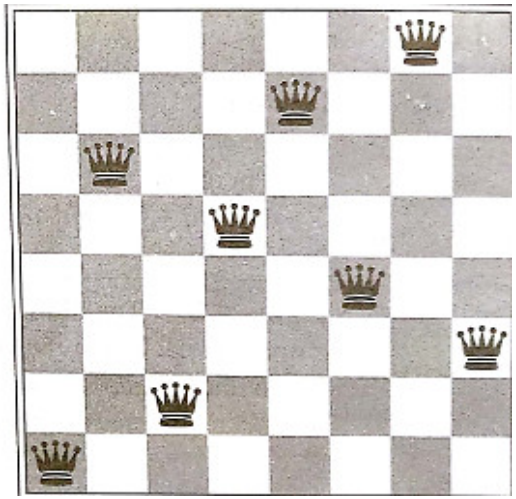
Algorithm 1: Hill-Climbing(problem)

```
1  Returned a state of problem that is local maximum.
2   current ← Make-Node (problem.Initial-State)
3  while not terminate do
4      neighbor ← a highest valued successor of current
5      if neighbor.Value ≤ current.Value then
6          return current.State
7      current ← neighbor
```

Hill Climbing Search

- Consider 8-queens problem
- In **complete-state formulation** has 8-queens in each state (one per column)
- Successor of a state are all possible states generated by moving a single queen to another square (each queen have 7 moves) so $8 \times 7 = 56$ successors.
- Heuristic cost function h is the number of pairs of queens that are attacking each other ($h = 17$ to $h = 1$)





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Sometime algorithm reaches to a state where no further progress could be made.

Due to

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2. Ridges
3. Plateaux

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Allow **sideway moves** in this scenario

How many sideway moves? ∞
Put a limit on this (say 100) Now it solves 94% problems. But, takes 21 steps to solve and 64 for failure on an average.

Hill Climbing Variants

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Instead of the best one, choose a path with probability of its steepness (converge slowly).

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Random-restart hill climbing

if probability of success is p then run $1/p$ number of times.

- For our previous case [a] it needs 7 iterations.
- and 22 steps

[a] With random start 8-queens state, hill climbing gets stuck 86% of time.

Simulated Annealing (Metallurgy approach)

- Simulated annealing is hill climbing combined with **random walk**
- Step size is gradually reduced
- First Applied around 1980, for VLSI layout problem

Algorithm 2: Simulated-Annealing(problem, schedule)

```
1 current ← Make-Node (problem.Initial-State)
2 for t = 1 to ∞ do
3     T ← schedule(t)
4     if T = 0 then return current
5     next ← a randomly selected successor of current
6     ΔE ← next.Value - current.Value
7     if ΔE > 0 then current ← next
8     else current ← next with probability e^(ΔE/T)
```

Genetic Algorithms

Evolution

- Recall Darwin's theory of evolution: "*Survival of the fittest*" [2]

[2] Images taken from various sources on Internet

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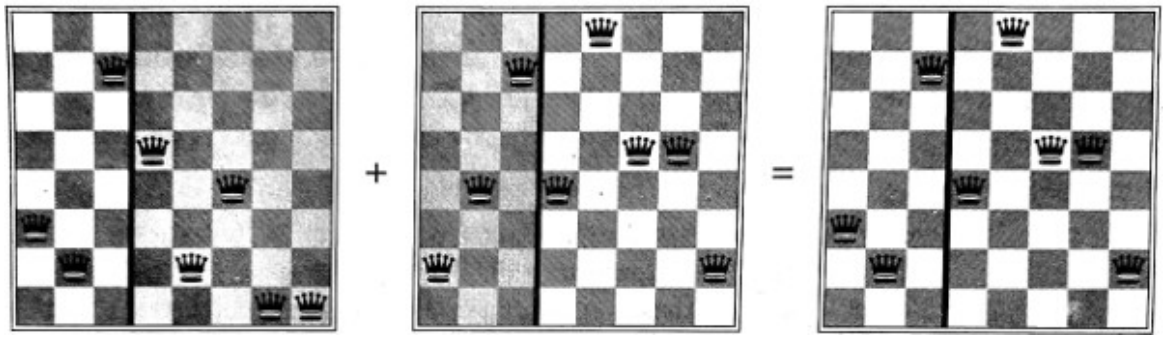
Is it true? Let's formulate and try..



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Without **guarantee**, GA often finds an object of high fitness

Genetic Algorithms

Algorithm 3: GA (`Fitness`, F_{th}, p, r, m)

1 P $\leftarrow$ generate p states at random
2 **while** $max(\texttt{Fitness}(h_1), \texttt{Fitness}(h_2), ..., \texttt{Fitness}(h_p)) < F_{th}$ **do**
3 **Select:** $(1 - r)p$ members of P
4 **Crossover:** on $(r \times p)/2$ pairs to produce two offspring
5 **Mutation:** randomly invert a bit of m percentage of population
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- Fitness function is typically a heuristic
- The fitness function is a criterion for ranking states to select states probabilistically for inclusion in the next generation population

Selection

- **Fitness proportionate**. Probability of selecting a state in next generation is

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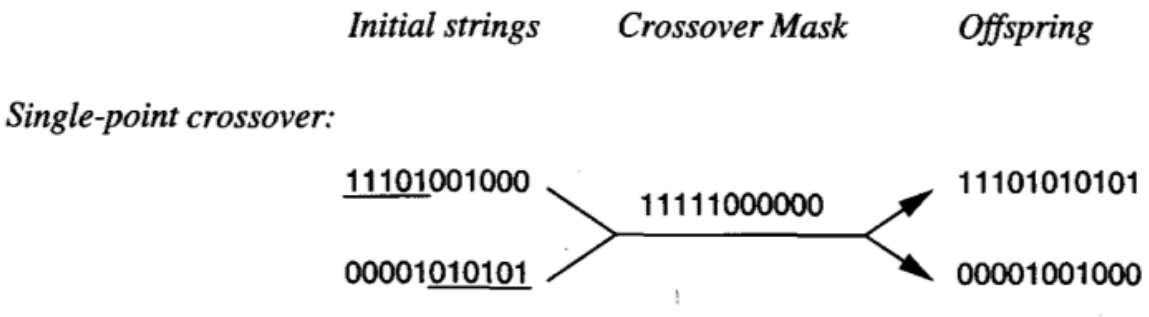
- **Elitist Model**. select a small proportion of the fittest candidates in current population intact into the next generation

Genetic Operators (crossover and mutation)



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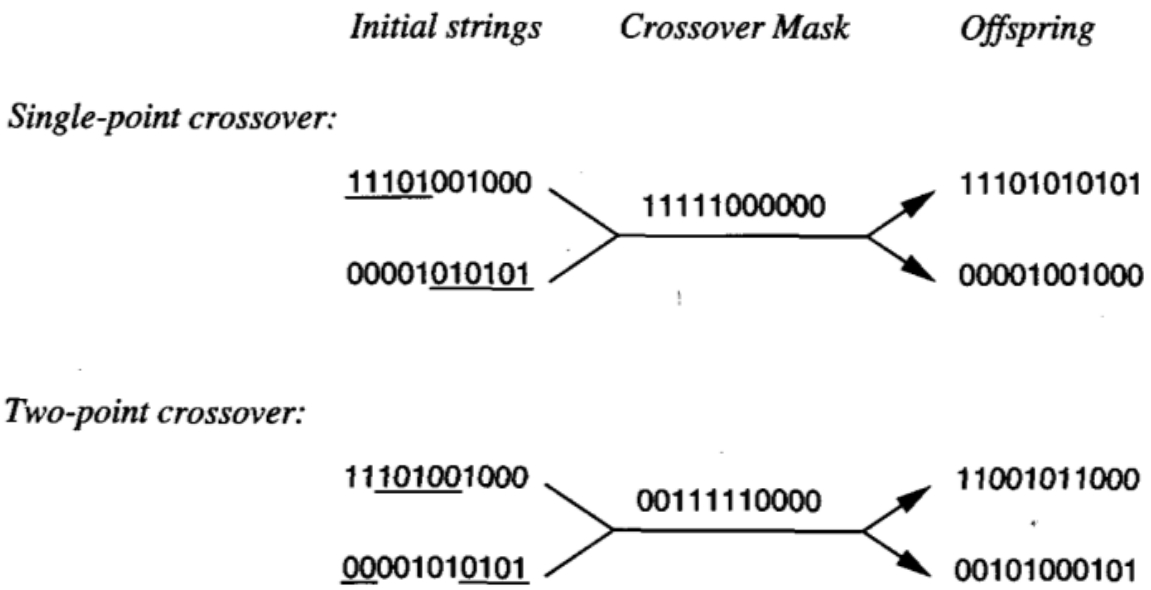

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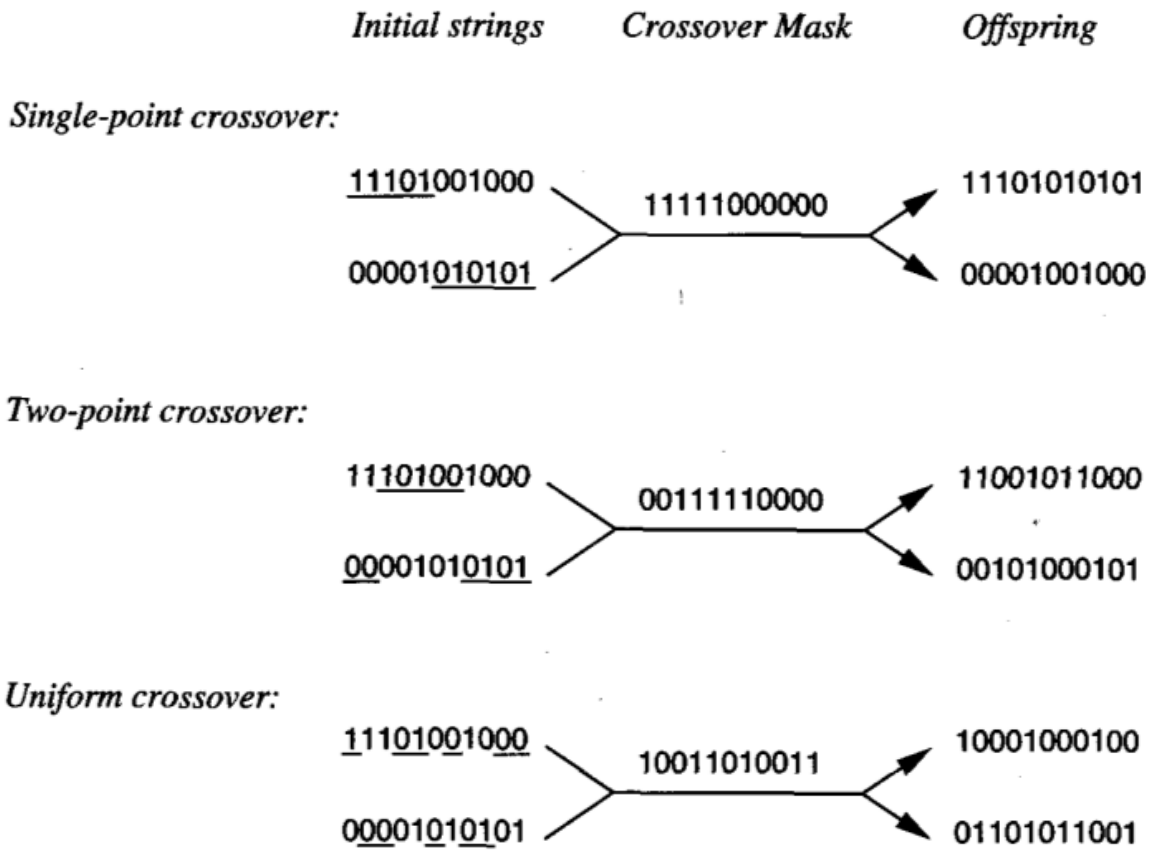

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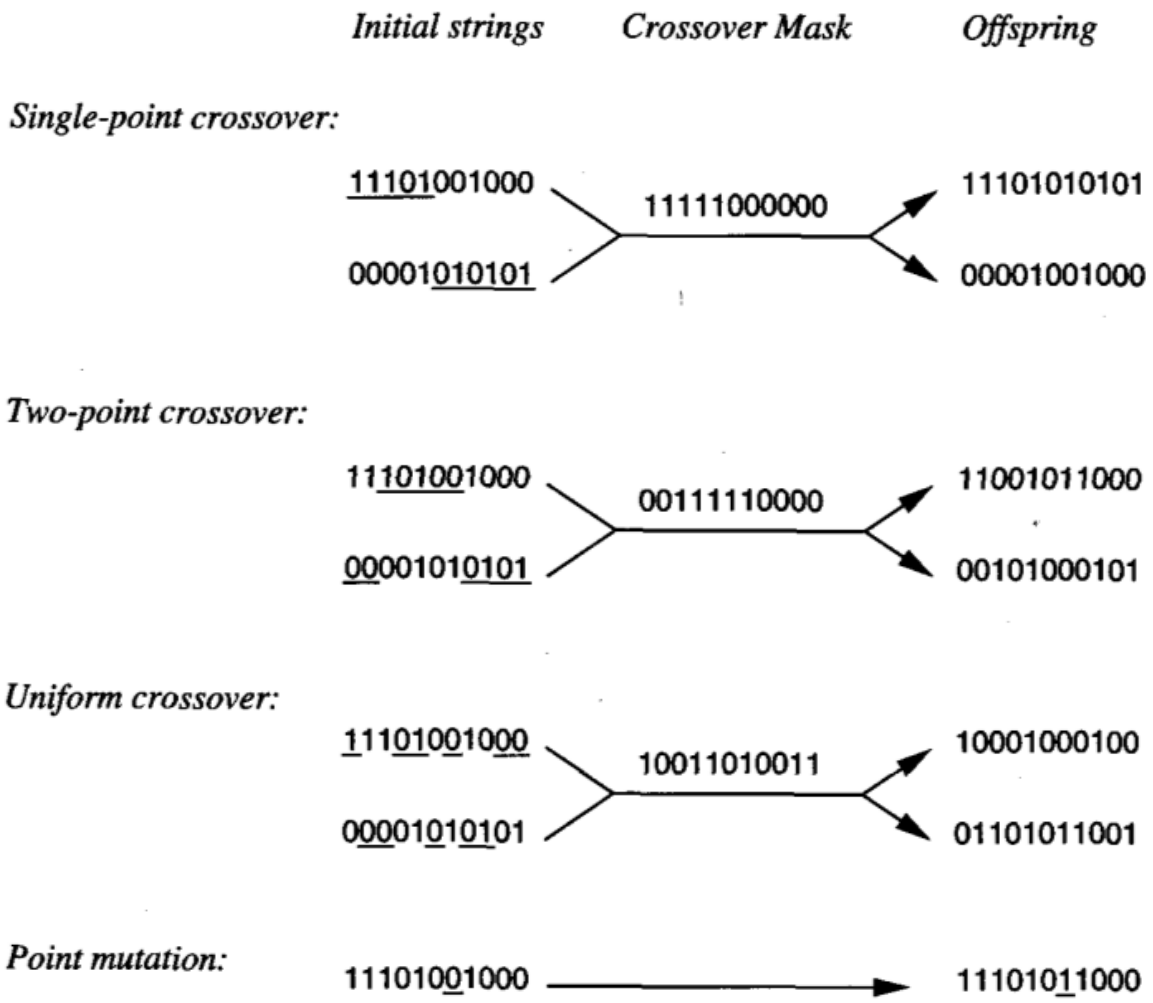

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$$\hat{u}(s,t) = \frac{\sum_{h \in s \cup p_t} f(h)}{m(s,t)}$$

- We know $Pr(h) = f(h)/(\sum f(h)) = f(h)/(n\bar{f}(t))$

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- Expected number of instances of s resulting from the n independent selection steps that create the entire new generation is just n times this probability. Therefore,

$$E[m(s,t+1)] = \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t)$$

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$$E[m(s, t+1)] = \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t)$$

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Thus, we can expect schema with above average fitness to be represented with increasing frequency on successive generations

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Full schema theorem thus provides a lower bound on the expected frequency of schema s, as follows

$$E[m(s,t+1)] \geq \frac{\hat{u}(s,t)}{\bar{f}(t)} m(s,t)\left(1 - p_c \frac{d(s)}{l-1}\right)(1-p_m)^{o(s)}$$

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Similar expression. More fit schemas will tend to grow in influence.

Thank You!

Thank you very much for your attention!

Queries ?

(Reference[3])

[3] 1) Book - *AIMA*, ch-04, Russell and Norvig. 2) Book - Machine Learning, ch-09, *Tom Mitchell*