IS-ZC444: Artificial Intelligence

Lecture-07: Beyond Classical Search

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Local Search in Continuous State

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- Minimize sum of distances of all the cities from its nearest airport

$$f(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{i=1}^{3} \sum_{c \in C_i} ((x_i - x_c)^2 + (y_i - y_c)^2)$$

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- Given locally correct values of $\frac{\partial f}{\partial x_1} = 2 \sum_{c \in C_1} (x_i - x_c)$ one can perform steepest-ascent using $x \leftarrow x + \alpha \bigtriangledown f$

Search with Non-Deterministic Actions

Non-Deterministic: not sure what would be the next state[1]

[1] Percepts would tell where have we reached.

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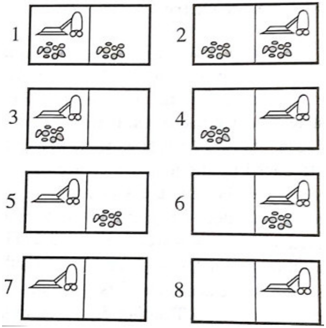

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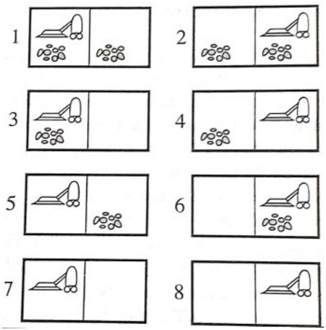

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- *suck* in 1, would lead $\{5,7\}$

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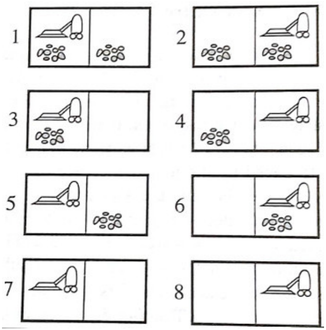

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[*suck*, **if** state=5 **then** [*right*,*suck* **else** []]

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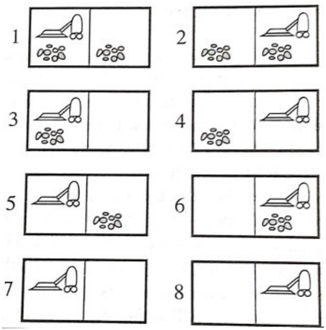

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- Search tree would contain some OR nodes and some AND nodes

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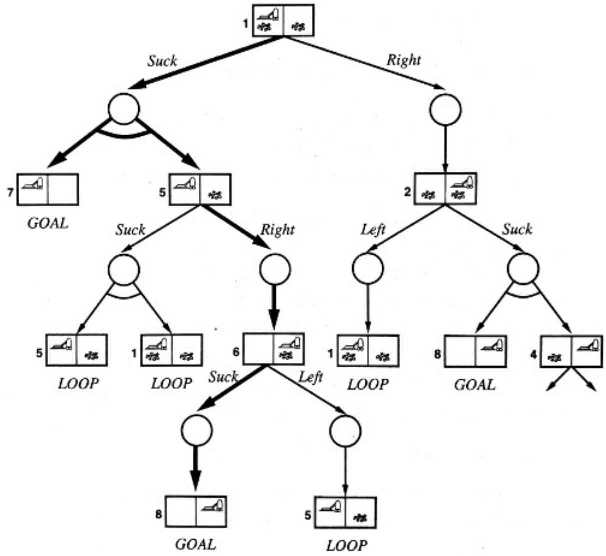
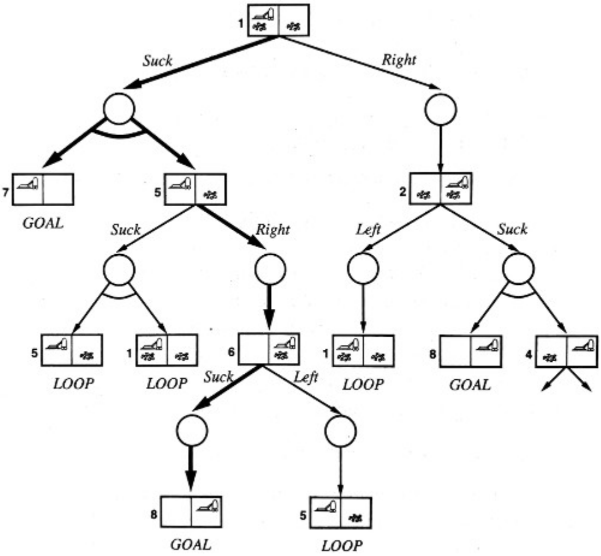
AND-OR Search Tree



AND-OR Search Tree



Solution

1. has goal node at every leaf
2. takes one action at each OR node
3. includes every outcome branch at each AND node

Searching with Partial Observations

When percepts do not suffice to pin down the exact state

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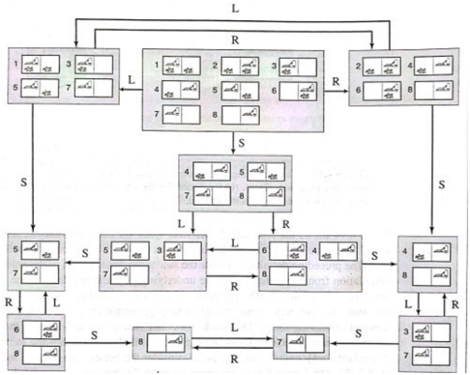
When percepts do not suffice to pin down the exact state

- **Sensor less.** consider [*right,suck,left,suck*] guarantees to reach in state 7 that is a goal state (traverses through belief states)
- All possible belief states may not be reachable (only 12 out of 2^8)



Online Search and Unknown Environment

Agent interleaves computation and action

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Take action $\rightarrow$ observe environment $\rightarrow$ compute next action

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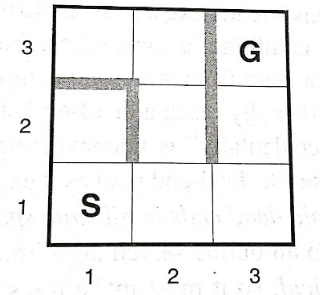

1. Consider following maze problem
2. A robot need to go from S to G
3. Knows nothing about the environment

Random-walk?

Online Search and Unknown Environment

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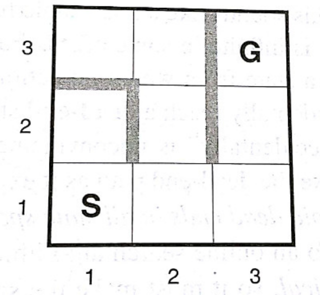

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Random-walk?

No algorithm can avoid dead-end in all state space

Adversarial Search (game)

Agents having conflicting goals in competitive multiagent environment

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- Chess has roughly branching factor 35, moves 50 so tree search space is $35^{100} = 10^{154}$ however, graph has 10^{40} nodes
- Finding optimal move is infeasible but, needs an ability to decide

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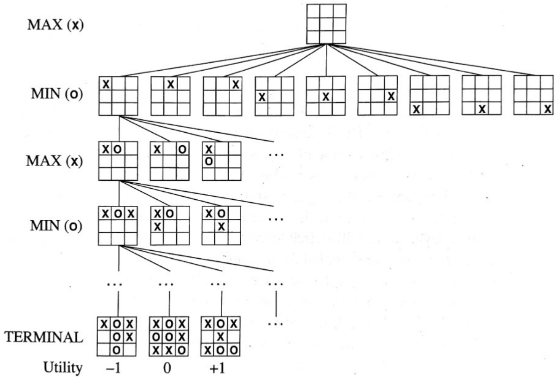
Game is between MAX and MIN (MAX moves first)

- S_0: the initial state
- PLAYER(s): defines which player has move to start
- ACTIONS(s): returns set of legal moves in a state
- RESULT(s, a):termination model defining result of a move
- TERMINAL_TEST(s): is true when game is over
- UTILITY(s, p): utility function defining reward (for chess +1,0,$1/2$)

Game Tree for tic-tac-toe



The search tree of the game has less than $9! = 362880$ nodes.

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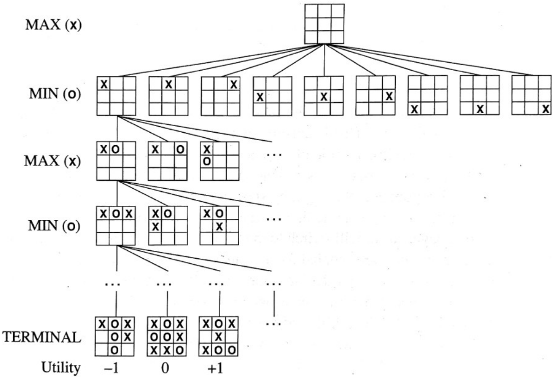
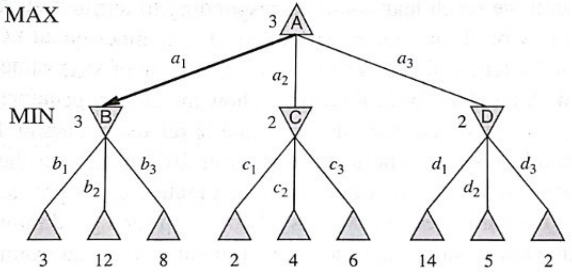


The search tree of the game has less than $9! = 362880$ nodes.

MAX must find a contingent **strategy**.

Analogous to AND-OR search (MAX plays OR and MIN plays AND)

Two half moves is one **ply**



[2] utility value for MAX of being in corresponding state (assuming then onwards both player play optimally)

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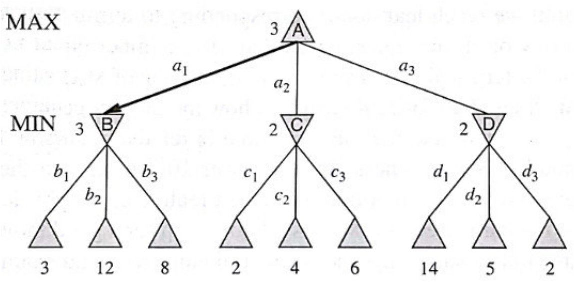

Given the game tree, optimal strategy can be determined from **minimax value** of each node.

$$MINIMAX(s) = \begin{cases} UTILITY(s) & \text{if } TERMINAL\_TEST(s) \\ max_{a \in Actions(s)} MINIMAX(RESULT(s, a)) & \text{if } PLAYER(s) = MAX \\ min_{a \in Actions(s)} MINIMAX(RESULT(s, a)) & \text{if } PLAYER(s) = MIN \end{cases}$$

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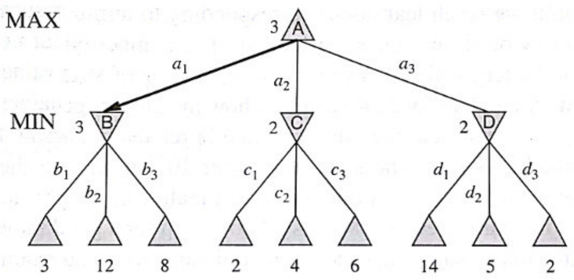

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Action a_1 is the optimal choice [2]
(essentially optimizing worst-case outcome for MAX)

[2] utility value for MAX of being in corresponding state (assuming then onwards both player play optimally)

MINIMAX Algorithm

Returns the action corresponding to best move

```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))

function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
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Recursion proceeds all the way down to the leaves.Time complexity $O(b^m)$ that is impractical but provides a basis of solution.

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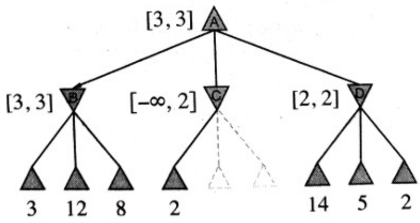

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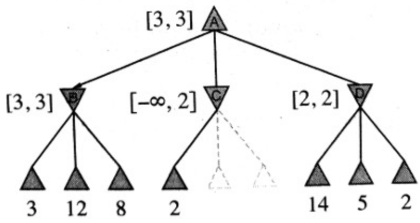

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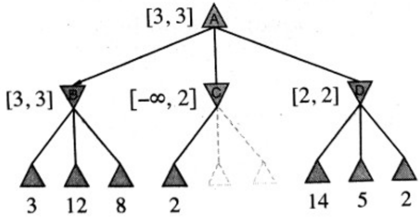

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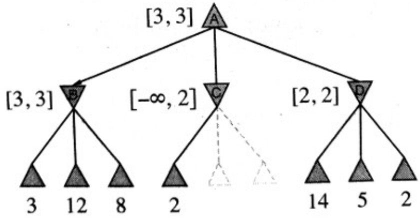

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= max(3, z, 2) where z=min(2,x,y)$\leq$ 2

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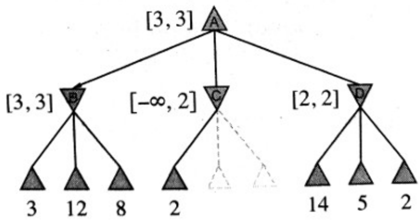

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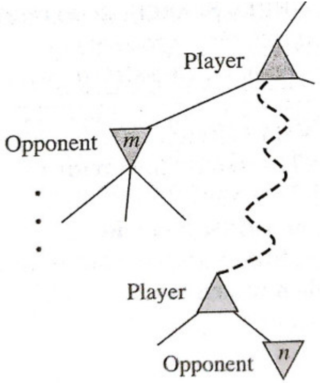
= 3

ALPHA-BETA Pruning

- Alpha-beta pruning can be applied to trees of any depth, and it is often possible to prune entire subtree rather than just leaves.



If *m* is better than *n* for player then we would never go to *n* in play

α	$=$	value of best choice (highest) found so far for MAX
β	$=$	value of best choice (lowest) found so far for MIN

ALPHA-BETA Pruning

```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, −∞, +∞)
  return the action in ACTIONS(state) with value v
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function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
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    if v ≤ α then return v
    β ← MIN(β, v)
  return v
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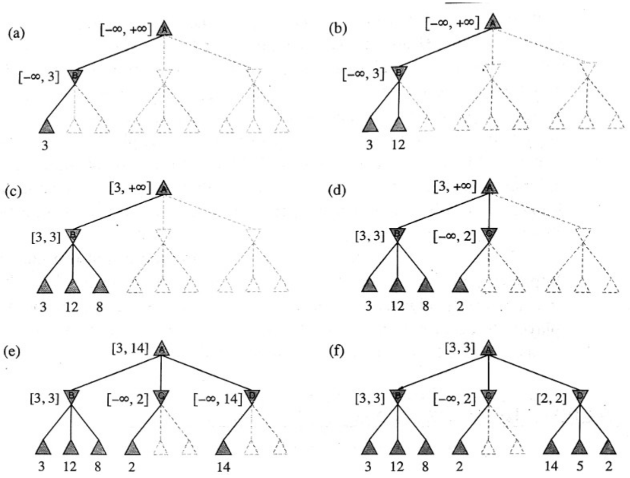
In-action: ALPHA-BETA Pruning



Thank You!

Thank you very much for your attention!

Queries ?

(Reference[3])

[3] 1) Book - *AIMA*, ch-04+05, Russell and Norvig.