IS-ZC444: Artificial Intelligence

Lecture-11: Logical Agents (Contd...)

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Recap: Propositional Logic

Propositions or **declarative sentences** can be true or false. Complex sentences could be formed by using $\neg$, $\wedge$, $\vee$ $\rightarrow$

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Natural Deduction applies proof rules on sentences ϕ_1, ϕ_2, ..., ϕ_n to get new ψ; we denote it as $\phi_1, \phi_2, ..., \phi_n \vdash \psi$

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$$\frac{\phi \qquad \psi}{\phi \wedge \psi} \wedge i \qquad \frac{\phi \wedge \psi}{\phi} \wedge e_1 \qquad \frac{\phi \wedge \psi}{\psi} \wedge e_2$$

$$\frac{\phi}{\neg\neg\phi} \neg\neg i \qquad \frac{\neg\neg\phi}{\phi} \neg\neg e \qquad \frac{\phi \qquad \phi \rightarrow \psi}{\psi} \rightarrow e$$

$$\frac{\phi \rightarrow \psi \qquad \neg\phi}{\neg\phi} MT \qquad \frac{\boxed{\begin{array}{c}\phi \\ \vdots \\ \psi\end{array}}}{\phi \rightarrow \psi} \rightarrow i$$

Box nesting is important

Backus-Naur Form (BNF)

There is a rule (syntax) to form sentences

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & True \mid False \mid P \mid Q \mid R \mid \ldots \\
ComplexSentence & \rightarrow & (\, Sentence \,) \mid [\, Sentence \,] \\
 & \mid & \neg\, Sentence \\
 & \mid & Sentence \wedge Sentence \\
 & \mid & Sentence \vee Sentence \\
 & \mid & Sentence \;\Rightarrow\; Sentence \\
 & \mid & Sentence \;\Leftrightarrow\; Sentence \\
\end{array}
$$

OPERATOR PRECEDENCE : $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

Rules for Disjunction

$$\frac{\phi}{\phi \vee \psi} \vee i_1 \qquad \frac{\psi}{\phi \vee \psi} \vee i_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi \\ \vdots \\ \chi\end{array}} \quad \boxed{\begin{array}{c}\psi \\ \vdots \\ \chi\end{array}}}{\chi} \vee e$$

Rules for Disjunction

$$\frac{\phi}{\phi \vee \psi} \vee i_1 \qquad \frac{\psi}{\phi \vee \psi} \vee i_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi \\ \vdots \\ x\end{array}} \quad \boxed{\begin{array}{c}\psi \\ \vdots \\ x\end{array}}}{x} \vee e$$

Show $p \vee q \vdash q \vee p$

Rules for Disjunction

$$\frac{\phi}{\phi \vee \psi} \vee i_1 \qquad \frac{\psi}{\phi \vee \psi} \vee i_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi \\ \vdots \\ x\end{array}} \quad \boxed{\begin{array}{c}\psi \\ \vdots \\ x\end{array}}}{x} \vee e$$

Show $p \vee q \vdash q \vee p$

1.	$p \vee q$	Premise

- Show $q \rightarrow r \vdash p \vee q \rightarrow p \vee r$

Rules for Disjunction

$$\frac{\phi}{\phi \vee \psi} \vee i_1 \qquad \frac{\psi}{\phi \vee \psi} \vee i_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi \\ \vdots \\ \chi\end{array}} \quad \boxed{\begin{array}{c}\psi \\ \vdots \\ \chi\end{array}}}{\chi} \vee e$$

Show $p \vee q \vdash q \vee p$

1.	$p \vee q$	Premise
2.	p	Assumption
3.	$q \vee p$	$\vee i_2$

- Show $q \to r \vdash p \vee q \to p \vee r$

Rules for Disjunction

$$\frac{\phi}{\phi \vee \psi} \vee i_1 \qquad \frac{\psi}{\phi \vee \psi} \vee i_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{matrix}\phi \\ \vdots \\ x\end{matrix}} \quad \boxed{\begin{matrix}\psi \\ \vdots \\ x\end{matrix}}}{x} \vee e$$

Show $p \vee q \vdash q \vee p$

1.	$p \vee q$	Premise
2.	p	Assumption
3.	$q \vee p$	$\vee i_2$
4.	q	Assumption
5.	$q \vee p$	$\vee i_1$

- Show $q \rightarrow r \vdash p \vee q \rightarrow p \vee r$

Rules for Disjunction

$$\frac{\phi}{\phi \vee \psi} \vee i_1 \qquad \frac{\psi}{\phi \vee \psi} \vee i_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi \\ \vdots \\ x\end{array}} \quad \boxed{\begin{array}{c}\psi \\ \vdots \\ x\end{array}}}{x} \vee e$$

Show $p \vee q \vdash q \vee p$

1.	$p \vee q$	Premise
2.	p	Assumption
3.	$q \vee p$	$\vee i_2$
4.	q	Assumption
5.	$q \vee p$	$\vee i_1$
6.	$q \vee p$	$\vee e$ 1, 3, 5

Rules for Disjunction

$$\frac{\phi}{\phi \vee \psi}\ \vee i_1 \qquad \frac{\psi}{\phi \vee \psi}\ \vee i_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{matrix}\phi \\ \vdots \\ x\end{matrix}} \quad \boxed{\begin{matrix}\psi \\ \vdots \\ x\end{matrix}}}{x}\ \vee e$$

Show $p \vee q \vdash q \vee p$

1.	$p \vee q$	Premise
2.	p	Assumption
3.	$q \vee p$	$\vee i_2$
4.	q	Assumption
5.	$q \vee p$	$\vee i_1$
6.	$q \vee p$	$\vee e$ 1, 3, 5

- Show $q \rightarrow r \vdash p \vee q \rightarrow p \vee r$

Rules for Negation

$$\frac{\bot}{\phi}\ \bot e \qquad \frac{\phi \qquad \neg\phi}{\bot}\ \neg e \qquad \frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\ \neg\, i$$

Rules for Negation

$$\frac{\bot}{\phi}\ \bot e \qquad \frac{\phi \quad \neg\phi}{\bot}\ \neg e \qquad \frac{\boxed{\begin{matrix}\phi\\ \vdots\\ \bot\end{matrix}}}{\neg\phi}\ \neg i$$

Show $\neg p \vee q \vdash p \to q$

Rules for Negation

$$\frac{\bot}{\phi}\ \bot e \qquad \frac{\phi \quad \neg\phi}{\bot}\ \neg e \qquad \frac{\boxed{\begin{matrix}\phi\\ \vdots\\ \bot\end{matrix}}}{\neg\phi}\ \neg i$$

Show $\neg p \vee q \vdash p \to q$

1.	$\neg p \vee q$	
2.	$\neg p$	Premise
3.	p	Assumption
4.	$\bot$	$\neg e$ 3,2
5.	q	$\bot e$ 4
6.	$p \to q$	$\to i$ 3,5

- Show $p \to q, p \to \neg q \vdash \neg p$

Rules for Negation

$$\frac{\bot}{\phi}\ \bot e \qquad \frac{\phi \quad \neg\phi}{\bot}\ \neg e \qquad \frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\ \neg i$$

Show $\neg p \vee q \vdash p \rightarrow q$

1.	$\neg p \vee q$			
2.	$\neg p$	Premise	q	Premise
3.	p	Assumption	p	Assumption
4.	$\bot$	$\neg e$ 3,2	q	Copy 2
5.	q	$\bot e$ 4	$p \rightarrow q$	$\rightarrow i$ 3,4
6.	$p \rightarrow q$	$\rightarrow i$ 3,5		

- Show $p \rightarrow q, p \rightarrow \neg q \vdash \neg p$

Rules for Negation

$$\frac{\bot}{\phi}\ \bot e \qquad\qquad \frac{\phi \qquad \neg\phi}{\bot}\ \neg e \qquad\qquad \frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\ \neg\, \mathrm{i}$$

Show $\neg p \lor q \vdash p \to q$

1.	$\neg p \lor q$			
2.	$\neg p$	Premise	q	Premise
3.	p	Assumption	p	Assumption
4.	$\bot$	$\neg e$ 3,2	q	Copy 2
5.	q	$\bot e$ 4	$p \to q$	$\to i$ 3,4
6.	$p \to q$	$\to i$ 3,5		
7.	$p \to q$			$\lor e$ 1,2-6

Rules for Negation

$$\frac{\bot}{\phi}\ \bot e \qquad \frac{\phi \quad \neg\phi}{\bot}\ \neg e \qquad \frac{\boxed{\begin{matrix}\phi\\ \vdots \\ \bot\end{matrix}}}{\neg\phi}\ \neg i$$

Show $\neg p \lor q \vdash p \rightarrow q$

1.	$\neg p \lor q$			
2.	$\neg p$	Premise	q	Premise
3.	p	Assumption	p	Assumption
4.	$\bot$	$\neg e$ 3,2	q	Copy 2
5.	q	$\bot e$ 4	$p \rightarrow q$	$\rightarrow i$ 3,4
6.	$p \rightarrow q$	$\rightarrow i$ 3,5		
7.	$p \rightarrow q$			$\lor e$ 1,2-6

- Show $p \rightarrow q, p \rightarrow \neg q \vdash \neg p$

Logical Equivalences

$(\alpha \wedge \beta) \quad \equiv \quad (\beta \wedge \alpha)$ Commutativity of $\wedge$

Logical Equivalences

$(\alpha \land \beta)$	=	$(\beta \land \alpha)$	Commutativity of $\land$
$(\alpha \lor \beta)$	=	$(\beta \lor \alpha)$	Commutativity of $\lor$

Logical Equivalences

$(\alpha \land \beta)$	$=$	$(\beta \land \alpha)$	Commutativity of $\land$
$(\alpha \lor \beta)$	$=$	$(\beta \lor \alpha)$	Commutativity of $\lor$
$(\alpha \land \beta) \land \gamma$	$=$	$\alpha \land (\beta \land \gamma)$	Associativity of $\land$

Logical Equivalences

$(\alpha \land \beta)$	$=$	$(\beta \land \alpha)$	Commutativity of $\land$
$(\alpha \lor \beta)$	$=$	$(\beta \lor \alpha)$	Commutativity of $\lor$
$(\alpha \land \beta) \land \gamma$	$=$	$\alpha \land (\beta \land \gamma)$	Associativity of $\land$
$(\alpha \lor \beta) \lor \gamma$	$=$	$\alpha \lor (\beta \lor \gamma)$	Associativity of $\lor$

Logical Equivalences

$(\alpha \wedge \beta)$	=	$(\beta \wedge \alpha)$	Commutativity of $\wedge$
$(\alpha \vee \beta)$	=	$(\beta \vee \alpha)$	Commutativity of $\vee$
$(\alpha \wedge \beta) \wedge \gamma$	=	$\alpha \wedge (\beta \wedge \gamma)$	Associativity of $\wedge$
$(\alpha \vee \beta) \vee \gamma$	=	$\alpha \vee (\beta \vee \gamma)$	Associativity of $\vee$
$\neg\neg\alpha$	=	α	Double negation elimination

Logical Equivalences

$(\alpha \wedge \beta)$	=	$(\beta \wedge \alpha)$	Commutativity of $\wedge$
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$(\alpha \wedge \beta) \wedge \gamma$	=	$\alpha \wedge (\beta \wedge \gamma)$	Associativity of $\wedge$
$(\alpha \vee \beta) \vee \gamma$	=	$\alpha \vee (\beta \vee \gamma)$	Associativity of $\vee$
$\neg\neg\alpha$	=	α	Double negation elimination
$\alpha \rightarrow \beta$	=	$\neg\beta \rightarrow \neg\alpha$	Contraposition

Logical Equivalences

$(\alpha \wedge \beta)$	$=$	$(\beta \wedge \alpha)$	Commutativity of $\wedge$
$(\alpha \vee \beta)$	$=$	$(\beta \vee \alpha)$	Commutativity of $\vee$
$(\alpha \wedge \beta) \wedge \gamma$	$=$	$\alpha \wedge (\beta \wedge \gamma)$	Associativity of $\wedge$
$(\alpha \vee \beta) \vee \gamma$	$=$	$\alpha \vee (\beta \vee \gamma)$	Associativity of $\vee$
$\neg\neg\alpha$	$=$	α	Double negation elimination
$\alpha \rightarrow \beta$	$=$	$\neg\beta \rightarrow \neg\alpha$	Contraposition
$\alpha \rightarrow \beta$	$=$	$\neg\alpha \vee \beta$	Implication Elimination

Logical Equivalences

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$(\alpha \wedge \beta) \wedge \gamma$	$=$	$\alpha \wedge (\beta \wedge \gamma)$	Associativity of $\wedge$
$(\alpha \vee \beta) \vee \gamma$	$=$	$\alpha \vee (\beta \vee \gamma)$	Associativity of $\vee$
$\neg\neg\alpha$	$=$	α	Double negation elimination
$\alpha \to \beta$	$=$	$\neg\beta \to \neg\alpha$	Contraposition
$\alpha \to \beta$	$=$	$\neg\alpha \vee \beta$	Implication Elimination
$\alpha \leftrightarrow \beta$	$=$	$(\alpha \to \beta) \wedge (\beta \to \alpha)$	Biconditional Elimination

Logical Equivalences

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$(\alpha \wedge \beta) \wedge \gamma$	=	$\alpha \wedge (\beta \wedge \gamma)$	Associativity of $\wedge$
$(\alpha \vee \beta) \vee \gamma$	=	$\alpha \vee (\beta \vee \gamma)$	Associativity of $\vee$
$\neg\neg\alpha$	=	α	Double negation elimination
$\alpha \rightarrow \beta$	=	$\neg\beta \rightarrow \neg\alpha$	Contraposition
$\alpha \rightarrow \beta$	=	$\neg\alpha \vee \beta$	Implication Elimination
$\alpha \leftrightarrow \beta$	=	$(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$	Biconditional Elimination
$\neg(\alpha \wedge \beta)$	=	$\neg\alpha \vee \neg\beta$	De Morgen

Logical Equivalences

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$(\alpha \vee \beta)$	$=$	$(\beta \vee \alpha)$	Commutativity of $\vee$
$(\alpha \wedge \beta) \wedge \gamma$	$=$	$\alpha \wedge (\beta \wedge \gamma)$	Associativity of $\wedge$
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$\alpha \rightarrow \beta$	$=$	$\neg\beta \rightarrow \neg\alpha$	Contraposition
$\alpha \rightarrow \beta$	$=$	$\neg\alpha \vee \beta$	Implication Elimination
$\alpha \leftrightarrow \beta$	$=$	$(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$	Biconditional Elimination
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$(\alpha \vee \beta) \vee \gamma$	$=$	$\alpha \vee (\beta \vee \gamma)$	Associativity of $\vee$
$\neg\neg\alpha$	$=$	α	Double negation elimination
$\alpha \rightarrow \beta$	$=$	$\neg\beta \rightarrow \neg\alpha$	Contraposition
$\alpha \rightarrow \beta$	$=$	$\neg\alpha \vee \beta$	Implication Elimination
$\alpha \leftrightarrow \beta$	$=$	$(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$	Biconditional Elimination
$\neg(\alpha \wedge \beta)$	$=$	$\neg\alpha \vee \neg\beta$	De Morgen
$\neg(\alpha \vee \beta)$	$=$	$\neg\alpha \wedge \neg\beta$	De Morgen
$\alpha \wedge (\beta \vee \gamma)$	$=$	$(\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$	Distribution of $\wedge$ on $\vee$

Logical Equivalences

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$\alpha \leftrightarrow \beta$	$=$	$(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$	Biconditional Elimination
$\neg(\alpha \wedge \beta)$	$=$	$\neg\alpha \vee \neg\beta$	De Morgen
$\neg(\alpha \vee \beta)$	$=$	$\neg\alpha \wedge \neg\beta$	De Morgen
$\alpha \wedge (\beta \vee \gamma)$	$=$	$(\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$	Distribution of $\wedge$ on $\vee$
$\alpha \vee (\beta \wedge \gamma)$	$=$	$(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$	Distribution of $\vee$ on $\wedge$

Soundness and Completeness

- Soundness: doing right
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Evaluate a legal system “guilty until proven innocent” and “innocent until proven guilty”

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What we want? both.

CNF, IMPL_FREE and NNF

Conjunctive normal form[1], implication free[2] and negative normal form[3]

Find CNF(NNF(IMPL_FREE(A)))

Where $A = \neg p \wedge q \rightarrow p \wedge (r \rightarrow q)$

[1] everything is conjunctions of disjunction
[2] no $\rightarrow$
[3] no double negation

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Conjunctive normal form[1], implication free[2] and negative normal form[3]

Find CNF(NNF(IMPL_FREE(A)))

Where $A = \neg p \wedge q \rightarrow p \wedge (r \rightarrow q)$

$$\neg(\neg p \wedge q) \vee (p \wedge (\neg r \vee q))$$
$$(p \vee \neg q) \vee (p \wedge (\neg r \vee q))$$
$$(p \vee \neg q \vee p) \vee (p \wedge \neg q \wedge \neg r \vee q)$$

[1] everything is conjunctions of disjunction

[2] no $\rightarrow$

[3] no double negation

Horn Clause

Formula that can be generated by H

$$\begin{aligned} P &::= \bot \mid \top \mid p \mid q \mid r \mid \ldots \\ A &::= P \mid P \wedge A \\ C &::= A \rightarrow P \\ H &::= C \mid C \wedge H \end{aligned} \tag{1}$$

Horn Clause

Formula that can be generated by H

$$
\begin{aligned}
P &::= \bot \mid \top \mid p \mid q \mid r \mid \ldots \\
A &::= P \mid P \wedge A \\
C &::= A \rightarrow P \\
H &::= C \mid C \wedge H
\end{aligned} \tag{1}
$$

Satisfiability

1. It marks $\top$ if it occurs in that list.
2. If there is a conjunct $P_1 \wedge P_2 \wedge \cdots \wedge P_{k_i} \rightarrow P'$ of ϕ such that all P_j with $1 \leq j \leq k_i$ are marked, mark P' as well and go to 2. Otherwise (= there is no conjunct $P_1 \wedge P_2 \wedge \cdots \wedge P_{k_i} \rightarrow P'$ such that all P_j are marked) go to 3.
3. If $\bot$ is marked, print out 'The Horn formula ϕ is unsatisfiable.' and stop. Otherwise, go to 4.
4. Print out 'The Horn formula ϕ is satisfiable.' and stop.

Horn Clause

Formula that can be generated by H

$$
\begin{aligned}
P &::= \bot \mid \top \mid p \mid q \mid r \mid \ldots \\
A &::= P \mid P \wedge A \\
C &::= A \rightarrow P \\
H &::= C \mid C \wedge H
\end{aligned} \tag{1}
$$

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3. If $\bot$ is marked, print out 'The Horn formula ϕ is unsatisfiable.' and stop. Otherwise, go to 4.
4. Print out 'The Horn formula ϕ is satisfiable.' and stop.

(a) $(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (\top \rightarrow r) \wedge (\top \rightarrow q) \wedge (u \rightarrow s) \wedge (\top \rightarrow u)$
(b) $(p \wedge q \wedge w \rightarrow \bot) \wedge (t \rightarrow \bot) \wedge (r \rightarrow p) \wedge (\top \rightarrow r) \wedge (\top \rightarrow q) \wedge (r \wedge u \rightarrow w) \wedge (u \rightarrow s) \wedge (\top \rightarrow u)$
(c) $(p \wedge q \wedge s \rightarrow p) \wedge (q \wedge r \rightarrow p) \wedge (p \wedge s \rightarrow s)$
(d) $(p \wedge q \wedge s \rightarrow \bot) \wedge (q \wedge r \rightarrow p) \wedge (\top \rightarrow s)$
(e) $(p_5 \rightarrow p_{11}) \wedge (p_2 \wedge p_3 \wedge p_5 \rightarrow p_{13}) \wedge (\top \rightarrow p_5) \wedge (p_5 \wedge p_{11} \rightarrow \bot)$
(f) $(\top \rightarrow q) \wedge (\top \rightarrow s) \wedge (w \rightarrow \bot) \wedge (p \wedge q \wedge s \rightarrow \bot) \wedge (v \rightarrow s) \wedge (\top \rightarrow r) \wedge (r \rightarrow p)$

Thank You!

Thank you very much for your attention!

Queries ?

(Reference[4])

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[4] 1) Book - *AIMA*, ch-07, Russell and Norvig. 2) Book - *Logic in CS*, ch-01, Mitchel Huth and Mark Ryan.