# IS/SE/SS ZC444: Artificial Intelligence

# 16 HMM, Neural Network ACO, PSO, Fairness

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## Markov Modal

- Andrev Markav: A canonical probabilistic model for temporal or sequential data. $X_0 \xrightarrow{A} X_1 \xrightarrow{A} ... \xrightarrow{A} X_n$
- Future is independent of past given the present. Assumption is that the present state encode all the history
- Order specifies how many evidences are important. Order three Markov Modal takes last three data
- iid[1] don't work.
- Temporal data, weather prediction, speech recognition, automatic music generation and handwriting recognition are some of the few applications

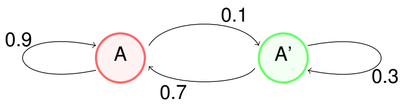
**Example:**
Suppose a company selling a product A (has market share of 20%), launches a advertise campaign that is expected to retain 90% old customers and attract 70% new. What maximum market share the product A can get?

[1] independent and identically distributed



## Markov Modal

**Transition diagram**




**Initial State**

$$S_0 = \begin{bmatrix} 0.2 & 0.8 \end{bmatrix}$$

**Transition matrix**

$$A = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix}$$

- $S_0 = \begin{bmatrix} 0.2 & 0.8 \end{bmatrix}$
- $S_1 = S_0 \times A = \begin{bmatrix} 0.74 & 0.26 \end{bmatrix}$
- $S_2 = S_1 \times A = \begin{bmatrix} 0.848 & 0.152 \end{bmatrix}$
- $S_3 = S_2 \times A = \begin{bmatrix} 0.8696 & 0.1304 \end{bmatrix}$



## Is it going to saturate?

**Stationary matrix**

$$\begin{bmatrix} a & b \end{bmatrix} \times A = \begin{bmatrix} a & b \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \times \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix} = \begin{bmatrix} a & b \end{bmatrix}$$

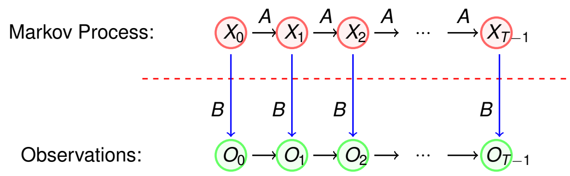
what are a and b? 0.875 and 0.125

- Does it always happen? No, only if matrix is **regular**
- When some power of the matrix has all positive values
- Which of these are regular?

$$\begin{bmatrix} 0.3 & 0.7 \\ 0.1 & 0.9 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0.2 & 0.8 \\ 1 & 0 \end{bmatrix}$$



## Hidden Markov Modal (HMM)




Assume we observe news coverage (S/M/L) of some article, to know whether a day was Hot or Cold?

$$B = \begin{array}{c c} & \begin{array}{ccc} S & M & L \end{array} \\ \begin{array}{c} H \\ C \end{array} & \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} \end{array} \qquad A = \begin{array}{c c} & \begin{array}{cc} H & C \end{array} \\ \begin{array}{c} H \\ C \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$



## Hidden Markov Modal (HMM)

$$B = \begin{array}{c c} & \begin{array}{ccc} S & M & L \end{array} \\ \begin{array}{c} H \\ C \end{array} & \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} \end{array} \qquad A = \begin{array}{c c} & \begin{array}{cc} H & C \end{array} \\ \begin{array}{c} H \\ C \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

- Assume initial configuration for H and C be $\pi = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}$
- And let observations be $S, M, S, L$
- Then what is $P(HHCC)$ ?
$0.6 \times 0.1 \times (0.7 \times 0.4) \times (0.3 \times 0.7) \times (0.6 \times 0.1) = 0.000212$

## Hidden Markov Modal (HMM)

| State | Probability | Normalized Probability |
|---|---|---|
| HHHH | 0.000412 | 0.042787 |
| HHHC | 0.000035 | 0.003635 |
| HHCH | 0.000706 | 0.073320 |
| HHCC | 0.000212 | 0.022017 |
| HCHH | 0.000050 | 0.005193 |
| HCHC | 0.000004 | 0.000415 |
| HCCH | 0.000302 | 0.031364 |
| HCCC | 0.000091 | 0.009451 |
| CHHH | 0.001089 | 0.114031 |
| CHHC | 0.000094 | 0.009762 |
| CHCH | 0.001882 | 0.195451 |
| CHCC | 0.000562 | 0.058573 |
| CCHH | 0.000470 | 0.048811 |
| CCHC | 0.000040 | 0.004154 |
| CCCH | 0.002822 | 0.293073 |
| CCCC | 0.000847 | 0.087963 |

Optimum state sequence

- In dynamic programming is CCCH
- HMM choses most probable symbol at each position. (by summation)

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(H) | 0.188182 | 0.519576 | 0.228788 | 0.804029 |
| P(C) | 0.811818 | 0.480424 | 0.771212 | 0.195971 |

Optimum state sequence in HMM is ? CHCH

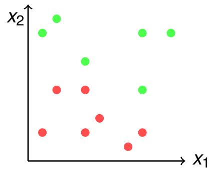


## Linear Classification

Consider Following data

| $x_1$ | $x_2$ | y |
|---|---|---|
| 1 | 9 | green |
| 10 | 9 | green |
| 4 | 7 | green |
| 4 | 5 | red |
| 5 | 3 | red |
| 8 | 9 | green |
| 4 | 2 | red |
| 2 | 5 | red |
| 7 | 1 | red |
| 2 | 10 | green |
| 8 | 5 | green |
| 1 | 2 | red |
| 8 | 2 | red |

Data is in 2D, so let us visualize




- Data looks linearly separable
- What is the decision boundary?

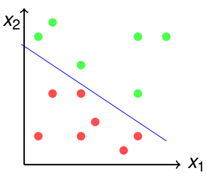
Many Possibilities, such as

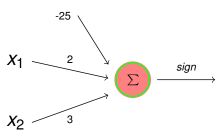
if ($2x_1 + 3x_2 - 25 > 0$) it is green otherwise red



## What about this arrangement?

With chosen *decision boundary* $2x_1 + 3x_2 - 25 = 0$




- This illustration is called as **perceptron**
- Provides a graphical way to represent the linear boundary
- Values 3, 2, -25 are its parameters or weights

Given a data

"How to find appropriate parameters?" is an important issue



## Perceptron Training Rule

Different algorithms may converge to different acceptable hypotheses

**Algorithm 1:** Perceptron training rule

Begin with random weights $w$
**repeat**
  **for** *each misclassified example* **do**
    $w_i = w_i + \eta(t - o)x_i$
**until** *all training examples are correctly classified*;
**return** $w$

- **Why would this strategy converge?**
  1. Weight does not change when classification is correct
  2. If perceptron outputs -1 when target is +1: weight increases ↑
  3. If perceptron outputs +1 when target is -1: weight decreases ↓

Conversion with perceptron training rule is subject to linear separability of training example and appropriate $\eta$



## Example

Consider the same data

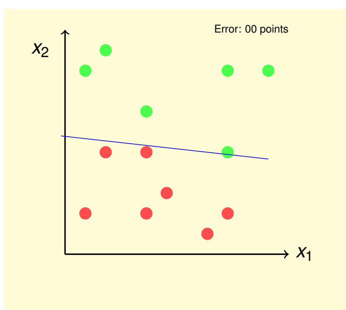
| $x_1$ | $x_2$ | y |
|---|---|---|
| 1 | 9 | green |
| 10 | 9 | green |
| 4 | 7 | green |
| 4 | 5 | red |
| 5 | 3 | red |
| 8 | 9 | green |
| 4 | 2 | red |
| 2 | 5 | red |
| 7 | 1 | red |
| 2 | 10 | green |
| 8 | 5 | green |
| 1 | 2 | red |
| 8 | 2 | red |

$\eta = 0.01$

| | |
|---|---|
| w0=0.500, w1=0.500, w2=0.500 | err=7 |
| w0=0.360, w1=-0.120, w2=0.100 | err=6 |
| w0=0.300, w1=-0.180, w2=0.060 | err=5 |
| w0=0.240, w1=-0.140, w2=0.140 | err=4 |
| w0=0.180, w1=-0.200, w2=0.100 | err=5 |
| w0=0.120, w1=-0.160, w2=0.180 | err=4 |
| w0=0.080, w1=-0.060, w2=0.180 | err=5 |
| w0=0.020, w1=-0.120, w2=0.140 | err=4 |
| w0=-0.040, w1=-0.180, w2=0.100 | err=5 |
| w0=-0.100, w1=-0.140, w2=0.180 | err=4 |
| w0=-0.140, w1=-0.040, w2=0.180 | err=5 |
| w0=-0.200, w1=-0.100, w2=0.140 | err=3 |
| w0=-0.260, w1=-0.160, w2=0.100 | err=4 |
| w0=-0.320, w1=-0.120, w2=0.180 | err=3 |
| w0=-0.360, w1=-0.020, w2=0.180 | err=3 |
| w0=-0.420, w1=-0.080, w2=0.140 | err=2 |
| w0=-0.420, w1=-0.080, w2=0.240 | err=2 |
| Fourteen more iterations | |
| w0=-0.900, w1=-0.020, w2=0.180 | err=1 |
| w0=-0.900, w1=-0.020, w2=0.240 | err=2 |
| w0=-0.920, w1=0.020, w2=0.220 | err=2 |
| w0=-0.960, w1=-0.020, w2=0.220 | err=3 |
| w0=-0.980, w1=0.020, w2=0.200 | err=2 |
| w0=-1.000, w1=0.060, w2=0.180 | err=2 |
| w0=-1.040, w1=0.020, w2=0.180 | err=0 |

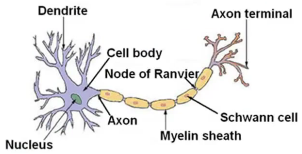


## Visual Interpretation




- Conversion is not gradual. (Error is NOT reducing monotonically)
- It is difficult to decide when to stop if data is not linearly separable

## Neural Network (NN)




- Cell, Axon, Synopses, Molicules, and Dendrites
- Humans have $10^{11}$ neurons, each connected to $10^4$ others, switches in $10^{-3}$ sec

**NN** is biologically motivated learning model that mimic human brain

- Started by *W. McCulloch* study on working of neurons in 1943
- MADALINE (1959), an adaptive filter that eliminates echoes on phone lines was the first neural network
- Popularity of Neural Network diminished in 90's but, due to advances in **processing power** and availability of **large data** it again became state-of-the-art



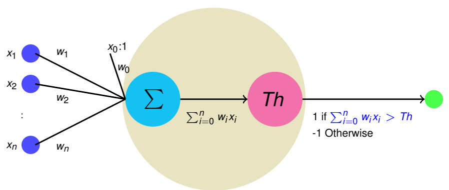
## Brief History

- 1872 Staining/Reticular Theory of Nervous Tissue
- 1943 McCulloch & Pitt (Neuron Model)
- 1947 Donald Hebb (Hebbian Learning)
- 1948 Nerbert (Cybernetics, optimal filter, feedback)
- 1954 Frank Rosenblatt (Perceptron)
- 1959 (MADALINE)
- 1962 Hubel & Wiesel (Visual Cortex Model)
- 1969 Minsky (Limitations of Perceptron)
- 1965 Frank Rosenblatt (MLP: Multi Layer Perceptron)
- 1986 (Backpropagation)
- 1989 Universal Approximation Theorem
- 1997 LSTM
- 1998 LeCun (ConvNet for MNIST digit)
- 2006 Unsupervised Pre Training
- 2010 Dahl. (Speech Recognition)
- 2012 AlexNet (ImageNet: Computer Vision) 26→16 ... →12% zfNet
- 2013 VGG →7.3
- 2014 Google LeNet →6.7
- 2015 ResNet →3.6
- 2017 DenseNet
- 2018 Transformers, U-Net segmentation



## A Single Perceptron

Perceptron representation



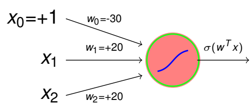

- A single perceptron can represent many boolean functions
- Any *m*-of-*n* function (at least *m* of the *n* inputs must be true) can be represented by perceptron. OR ($m$=1) and AND ($m$=$n$)

Two layer NN can represent any boolean function (Consider SOP)



## An Example

Consider a perceptron with output 0/1 as below




| $x_1$ | $x_2$ | Output |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

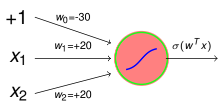
This perceptron computes logical AND

- $w_0$=-10 gives logical OR
- $w_0$=10, $w_1$=-20 with single input gives logical NOT
- XOR is not possible (MLP[2] can do it!)

[2] MLP (multi layer perceptron) with one-hidden-layer is *universal boolean function*, capable of expressing any truth table



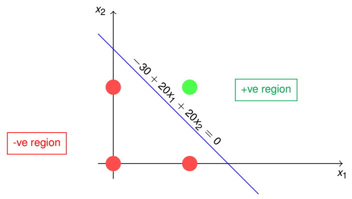
## Essentially it Represents A Decision Boundary




Provides positive classification if

$$-30 + 20x_1 + 20x_2 \geq 0$$

Represents a linear decision boundary



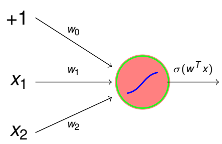



## An Example

Design a **perceptron** for

| $x_1$ | $x_2$ | Classification |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Let us assume following




We have following four equations

$$w_0 + w_1 \times (0) + w_2 \times (0) < 0 \quad (1)$$
$$w_0 + w_1 \times (0) + w_2 \times (1) < 0 \quad (2)$$
$$w_0 + w_1 \times (1) + w_2 \times (0) \geq 0 \quad (3)$$
$$w_0 + w_1 \times (1) + w_2 \times (1) < 0 \quad (4)$$

By (1) $w_0 < 0$ so let $w_0 = -1$
By (2) $w_0+w_2<0$ so let $w_2 = -1$
By (3) $w_0+w_1 \geq 0$ so let $w_1 = 1.5$
By (4) $w_0+w_1+w_2<0$ that is valid

$\boxed{\text{So } (w_0, w_1, w_2) = (-1, -1, 1.5)}$

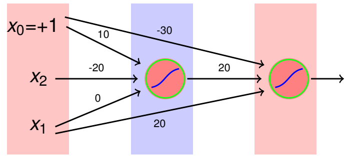
Other possibilities are also there

## An Example

Design a **neural network** for

| $x_1$ | $x_2$ | Classification |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| $x_1$ | $x_2$ | $\bar{x}_2$ | $(x_1)AND(\bar{x}_2)$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

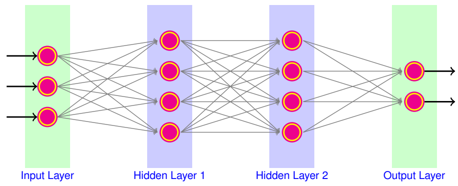



This arrangement is mostly avoided, as training is very challenging



## Neural Network

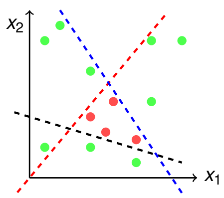
When neurons are interconnected in layers




- Number of layers may differ
- Nodes in each intermediate layers may also differ
- Multiple output neurons are used for different class
- **Two levels deep** NN can represent any boolean function

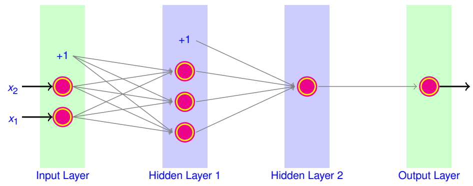


## More Example: Design NN for the following data




Whether it is green?

| Red-line | Blue-line | Black-line | Color |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |




Note: Weights and activation in subsequent layers add power to the model in terms of non linearity.



## Neural Network Applications

NN is appropriate for problems with the following characteristics:

- Instances are provided by many attribute-value pairs (more data)
- The target function output may be discrete-valued, real-valued, or a vector of several real or discrete valued attributes
- The training examples may contain errors
- Long training times are acceptable
- Fast evaluation of the target function may be required
- The ability of humans to understand the learned target function is not important



## Perceptron Training (delta rule)

When data is not linearly-separable, error fluctuates with parameter update so, it becomes difficult to decide when to stop

- Delta rule converges to a best-fit approximation of the target
- Uses gradient descent
- Consider unthresholded perceptron, $o(\vec{x}) = \vec{w}.\vec{x}$
- Training error is defined as

$$E(\vec{w}) = \frac{1}{2}\sum_{d \in D}(t_d - o_d)^2$$

- Gradient would specify direction of steepest increase $\nabla E(\vec{w}) = \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \dots, \frac{\partial E}{\partial w_n}\right]$
- Weights can be learned as $w_i = w_i - \eta\frac{\partial E}{\partial w_i}$
- It can be seen that $\frac{\partial E}{\partial w_i} = \sum_{d \in D}(t_d - o_d)(-x_{id})$



## Perceptron Training (delta rule)

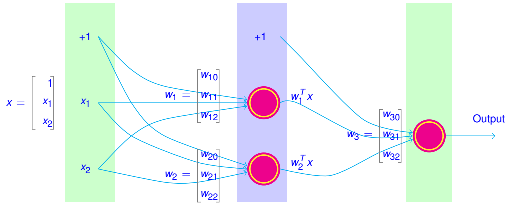
**Algorithm 2:** Gradient Descent (D, $\eta$)

```
Initialize w_i with random weights
repeat
    For each w_i, initialize △w_i = 0
    for each training example d ∈ D do
        Compute output o using model for d whose target is t
        For each w_i, update △w_i = △w_i + η(t−o)x_i
    For each w_i, set w_i = w_i + △w_i
until termination condition is met;
return w
```

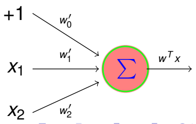
- A date item $d \in D$, is supposed to be multidimensional $d = (x_1, x_2, \dots, x_n, t)$
- Algorithm converges toward the minimum error hypothesis.
- Linear programming can also be an approach

## Linear Activation is Not Much Interesting

NN with perceptrons have limited capability, even with many layers




$$
\begin{aligned}
Output &= w_{30} \times 1 + w_{31} \times (w_1^T x) + w_{32} \times (w_2^T x) \\
&= w_{30} \times 1 + w_{31} \times [w_{10} \times 1 + w_{11} \times x_1 + w_{12} \times x_2] \\
&\quad + w_{32} \times [w_{20} \times 1 + w_{21} \times x_1 + w_{22} \times x_2] \\
&= (w_{30} + w_{31} w_{10} + w_{32} w_{20}) + (w_{31} w_{11} + w_{32} w_{21}) \times x_1 \\
&\quad + (w_{31} w_{12} + w_{32} w_{22}) \times x_2 \\
&= w'_0 + w'_1 \times x_1 + w'_2 \times x_2
\end{aligned}
$$

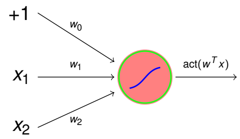


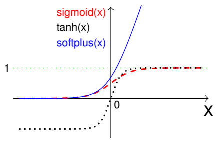
**Expression of single perceptron**

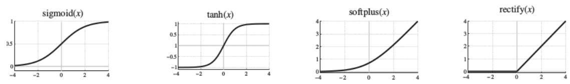


## Neuron

**Neuron** uses nonlinear activation functions (*sigmoid*, *tanh*, *ReLU*, *softplus etc.*) at the place of thresholding





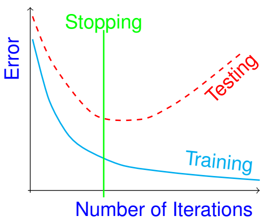




$h(x) = \frac{1}{1+\exp(-x)}$ $\quad h(x) = \frac{\exp(x)-\exp(-x)}{\exp(x)+\exp(-x)}$ $\quad h(x) = \log(1+\exp(x))$ $\quad h(x) = \max(0, x)$



## Generalization, Overfitting, and Stopping Criterion

Continue training until the error on the training examples falls below some predetermined threshold could be a poor strategy

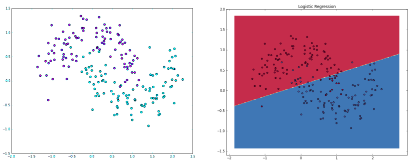


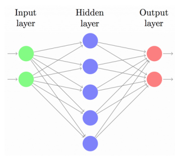

- Weight decay or use of validation set ($k$-fold ?) is suggested
- Input or output encoding can be used

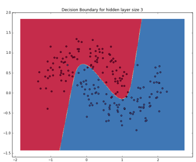


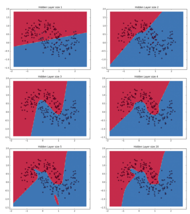
## Coding Example [3]

Consider two class data
Logistic Regression
Neural Network
Decision Boundary
Bigger hidden layer





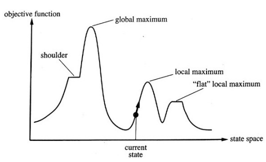



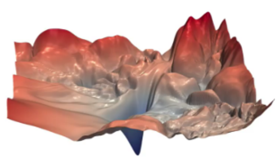


[3] https://github.com/dennybritz/nn-from-scratch/blob/master/nn-from-scratch.ipynb



## Optimization



We essentially need **convexity**

$$f(\alpha x_1 + (1-\alpha)x_2) \leq \alpha f(x_1) + (1-\alpha) f(x_2)$$

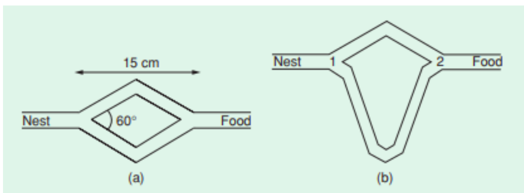
for all $\alpha \in (0, 1)$

It is used in weight update as $w_i = w_i - \alpha \frac{\partial J}{\partial w_i}$



## Ant Clony Optimization [4]

Swarm intelligence takes inspiration from the social behaviors of insects and of other animals for problem solving




Pheromone for probability,

[4] Ref-15420 Dorigo, Marco and Birattari, Mauro and Stutzle, Thomas. **Ant colony optimization**, IEEE computational intelligence magazine, 1(4), pp 28–39, 2006, IEEE

## Particle Swarm Optimization [5]

Population based stochastic algorithm for optimization of nonlinear functions

1. Initialize particles
2. Calculate fitness of all particle and maintain best-till-now
3. Who has highest best-till-now
4. Update particle locations in direction of global fit

Conceptually, it seems to lie somewhere between genetic algorithms and evolutionary programming. It is highly dependent on stochastic processes, like evolutionary programming. The adjustment toward pbest and gbest by the particle swarm optimizer is conceptually similar to the crossover operation utilized by genetic algorithms. It uses the concept of fitness, as all evolutionary computation paradigms.

[5] Ref-74943 Kennedy, James and Eberhart, Russell. **Particle swarm optimization**, International conference on neural networks, vol 4, pp 1942–1948, 1995, IEEE



## Fairness in Model

How to ensure that **data biases** and **model inaccuracies** do not treat individuals unfavorably on the basis of race, gender, disabilities, and sexual or political orientation?

- Fairness through unawareness
- Equalized odds

$$P(\hat{y}|a = 0, Y = y) = P(\hat{y}|a = 1, Y = y)$$

- Equalized opportunity

$$P(\hat{y} = 1|a = 0, Y = 1) = P(\hat{y} = 1|a = 1, Y = 1)$$

Correlation fallasy, overgeneraization, class imbalance.



## Interpretable/Explainable Model

Computers usually do not explain their predictions. How can we develop trust?

- We need causality, transferability, informativeness, fairness and ethical decision
- Transparency has three levels
  Simulatibility: one can do it on paper
  Decomposability: different parts
  Algorithmic transparency: convergence guarantees
- Decision tree and linear models



## Thank You!

**Thank you very much for your attention!**

**Queries ?**