CS F364: Design & Analysis of Algorithms

Lecture-kt10: Binary Search Tree (contd.) + Red-Black Trees

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Recap: Binary Search Tree

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- Basic operations on a binary search tree take time proportional to the height of the tree
- **Binary-search-tree property:** Let x be a node in a binary search tree. If y is a node in the
  - **left** subtree of x, then $y.key \leq x.key$
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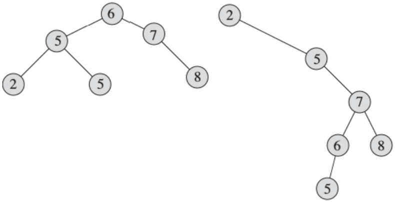
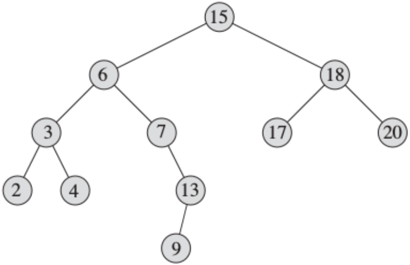


- We may safely assume all items to be different
- In order traversal is monotonous

Search in BST



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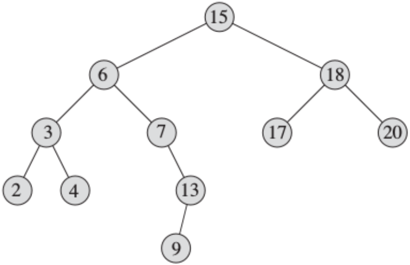
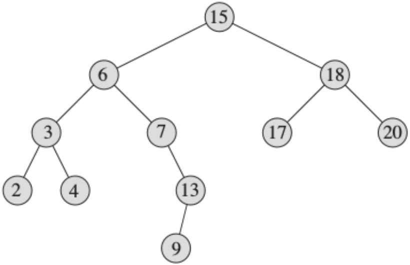


Algorithm 2: BST-Search(x,k)

if $x = nil$ *or* $k = x.key$ **then**
  return x
if $k < x.key$ **then**
  return BST-Search($x.left$,k)
else
  return BST-Search($x.right$,k)

Search in BST



Algorithm 3: BST-Search(x,k)

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if x = nil or k = x.key then
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```

While searching for 363, which can not be a search sequence

a. 2, 252, 401, 398, 330, 344, 397, 363.

b. 924, 220, 911, 244, 898, 258, 362, 363.

c. 925, 202, 911, 240, 912, 245, 363.

d. 2, 399, 387, 219, 266, 382, 381, 278, 363.

e. 935, 278, 347, 621, 299, 392, 358, 363.

Minimum Maximum and Successor

Algorithm 4: BST-Minimum(x)

while $x.left \neq nil$ **do**
  $x = x.left$
return x

Minimum Maximum and Successor

Algorithm 7: BST-Minimum(x)

while $x.left \neq nil$ **do**
  $x = x.left$
return x

Algorithm 8: BST-Maximum(x)

while $x.right \neq nil$ **do**
  $x = x.right$
return x

Minimum Maximum and Successor

Algorithm 10: BST-Minimum(x)

```
while x.left ≠ nil do
    x = x.left
return x
```

Algorithm 11: BST-Maximum(x)

```
while x.right ≠ nil do
    x = x.right
return x
```

Successor is next smallest number

Algorithm 12: BST-Successor(x)

```
if x.right ≠ nil then
    return BST-Minimum(x.right)
y = x.parent
while y ≠ nil and x = y.right do
    x = y
    y = x.parent
return y
```

Insertion in BST

- Tree has
  - root

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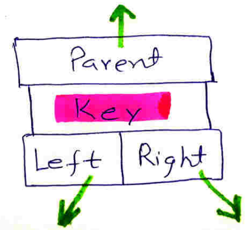
  - key
  - left
  - right
  - parent or p



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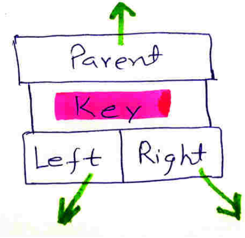

Algorithm 15: BST-Insert(T,z)

```
y = nil, x = T.root, z.left=nil, z.right=nil
while x ≠ nil do
    y=x
    if z.key < x.key then
        x=x.left
    else
        return x=x.right
z.parent = y
if y == nil then
    T.root = z
else if z.key < y.key then
    y.left=z
else
    y.right = z
```

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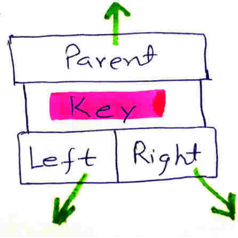

Algorithm 16: BST-Insert(T,z)

y = nil, x = T.root, z.left=nil, z.right=nil
while $x \neq nil$ **do**
    y=x
    if $z.key < x.key$ **then**
        x=x.left
    else
        return x=x.right
z.parent = y
if $y == nil$ **then**
    T.root = z
else if $z.key < y.key$ **then**
    y.left=z
else
    y.right = z

Time taken is proportional to height of tree

Deletion in BST

Algorithm 17: BST-Transplant(T,u,v)

if $u.parent = nil$ **then**
    $T.root = v$
else if $u = u.parent.left$ **then**
    $u.parent.left = v$
else
    $u.parent.right = v$
if $v \neq nil$ **then**
    $v.parent = u.parent$

Deletion in BST

Algorithm 19: BST-Transplant(T,u,v)

if $u.parent = nil$ **then**
  $T.root = v$
else if $u = u.parent.left$ **then**
  $u.parent.left = v$
else
  $u.parent.right = v$
if $v \neq nil$ **then**
  $v.parent = u.parent$

Algorithm 20: BST-Delete(T,z)

if $z.left = nil$ **then**
  BST-Transplant(T, z, z.right)
else if $z.right = nil$ **then**
  BST-Transplant(T, z, z.left)
else
  y = BST-Minimum(z.right) **if** $y.parent \neq z$ **then**
    BST-Transplant(T, y, z.right)
    y.right=z.right
    y.right.parent = y
  BST-Transplant(T, z, y)
  y.left=z.left
  y.left.parent = y

Time is proportional to tree height, (is $O(\log n)$ when build randomly)

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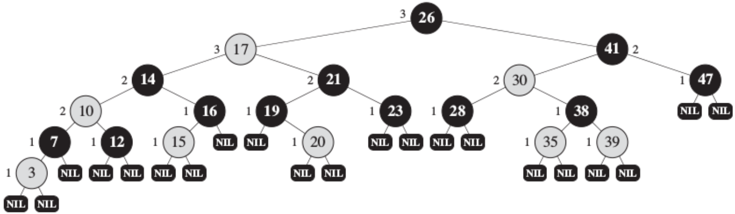

Height of Red-Black Trees

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3. Black-height of the root must be at least $h/2$; thus $n \geq 2^{h/2} - 1$ that gets the same.

Thank You!

Thank you very much for your attention!

Queries ?