CS F364: Design & Analysis of Algorithms

Lecture-kt08: Hashing

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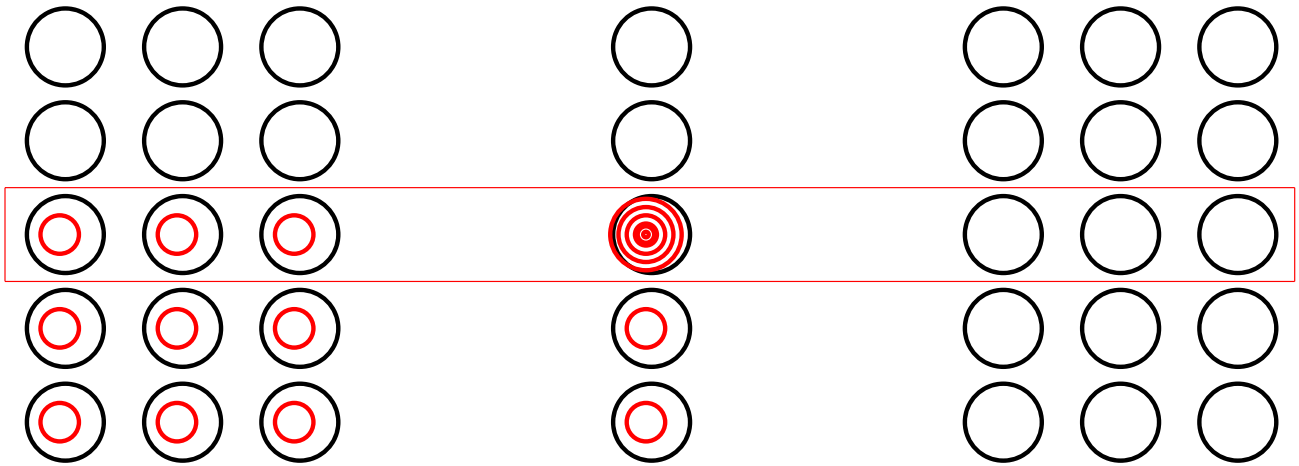

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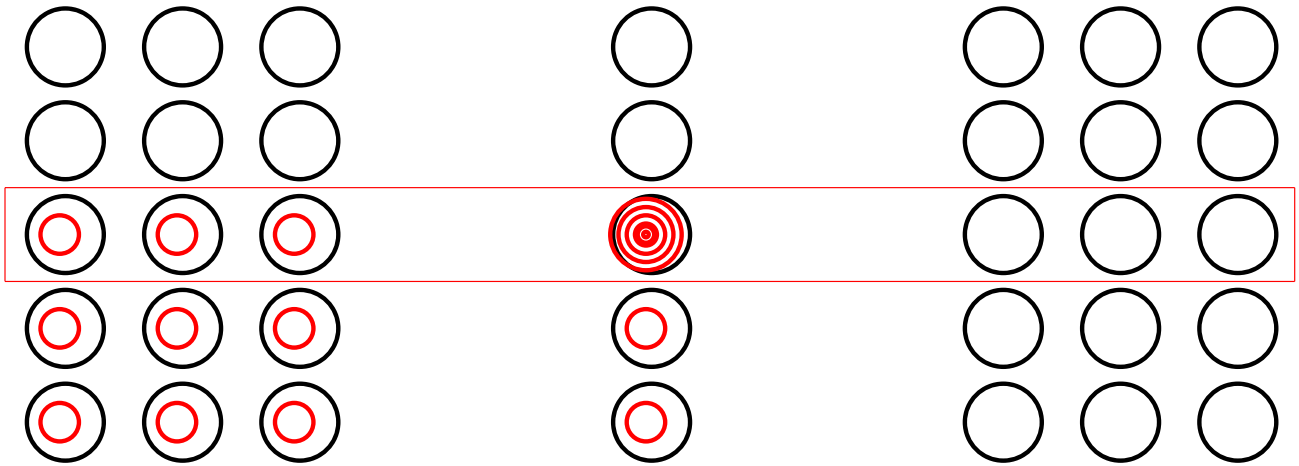

- Running time for order statics is $T(n) = T(7n/10) + O(n)$ that solves to $O(n)$ in worst case. ☺

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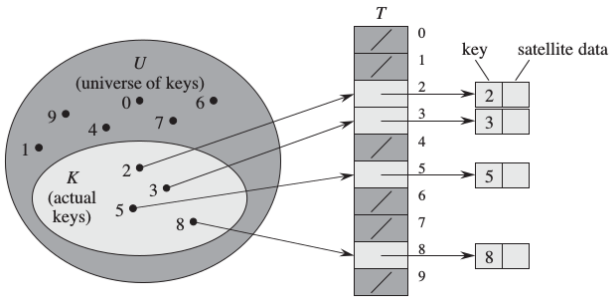

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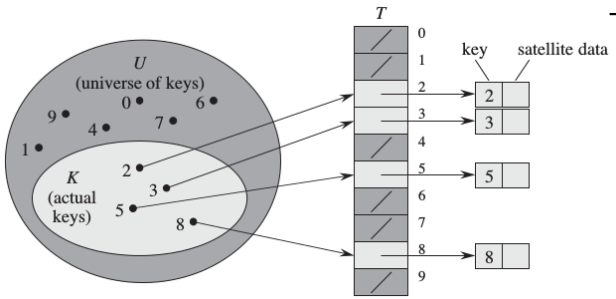

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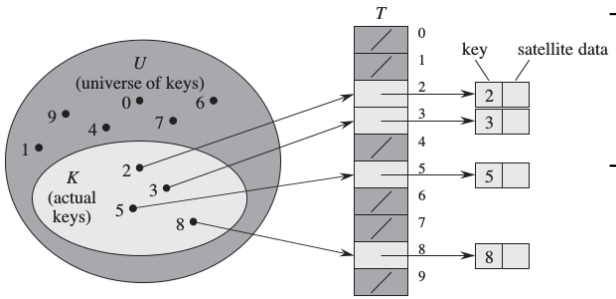


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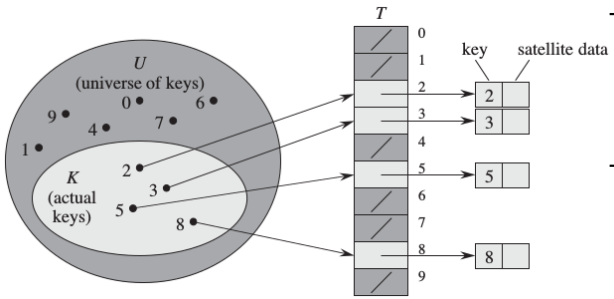

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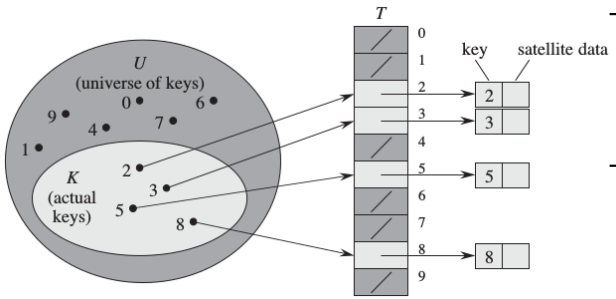

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All operations take $O(n)$ time

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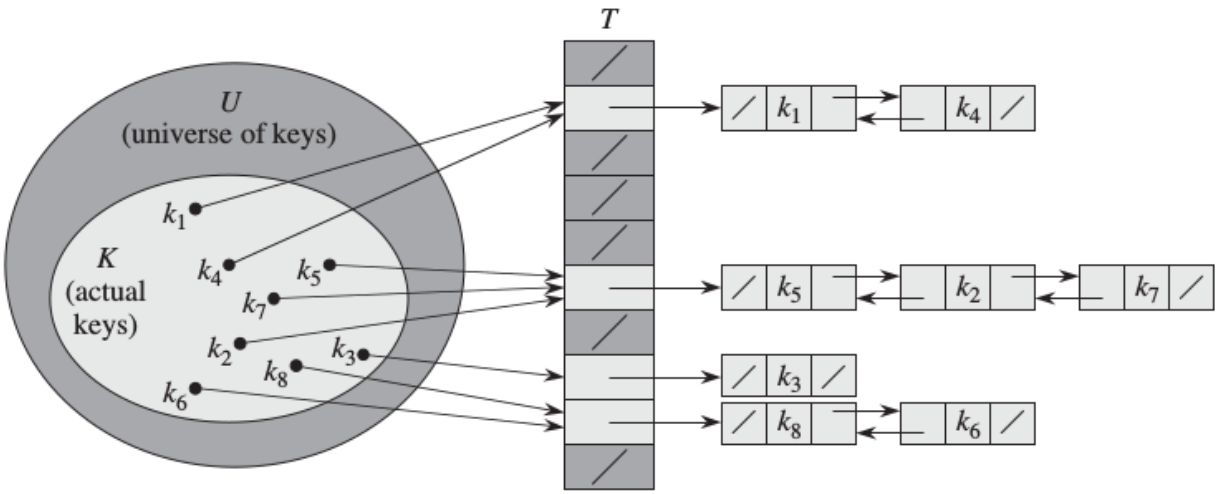

Dictionary operations in Hash Table

CHAINED-HASH-INSERT(T, x)

insert x at the head of list $T[h(x.key)]$

CHAINED-HASH-SEARCH(T, k)

search for an element with key k in list $T[h(k)]$

CHAINED-HASH-DELETE(T, x)

delete x from the list $T[h(x.key)]$

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- Successful and unsuccessful search takes $\Theta(\alpha + 1)$ time on an average

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- Therefore, choose the hash function randomly independently of the keys to be stored
- The approach is called **universal hashing**
- Let H be finite collection of hash function that map a given universe U of keys into the range $\{0, 1, 2, ..., m\}$. such a collection is said to be **universal** if for each pair of distinct keys $x, y \in U$, the number of hash functions $h \in H$ for which $h(x) = h(y)$ is precisely $|H|/m$.

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- Let $a =< a_0, a_1, a_2, ..., a_r >$ denote a sequence of $r+1$ elements chosen randomly from the set $\{0, 1, 2, ..., m\}$.
- We define m^{r+1} size H, having hash function $h_a \in H$ as

$$h_a(x) = \sum_{i=0}^{r} a_i x_i \bmod m$$

Proof: Assume $x_0 \neq y_0$ for distinct x, y. For any $< a_1, a_2, ..., a_r >$ there is exactly one a_0 satisfying $h(x) = h(y)$; this a_0 is the solution to $a_0(x_0 - y_0) \equiv -\sum_{i=1}^{r} a_i(x_i - y_i) (\text{mod } m)$. Since m is prime, the nonzero quantity $x_0 - y_0$ has multiplicative inverse modulo m, and thus there is a unique solution for a_0 modulo m. Therefore, x and y collides on m^r values of a, as they collide exactly once for each possible values of $< a_1, a_2, ..., a_r >$. Since $|a| = m^{r+1}$, the key x and y collide with probability exactly $m^r/m^{r+1} = 1/m$. Therefore H is universal.

Thank You!

Thank you very much for your attention!

Queries ?