# CS-F415: Data Mining

# 10 Boosting Bagging

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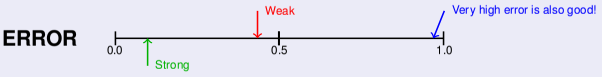
## Classification

- Binary classification approaches
  - Linear classifiers
  - Quadratic classifiers
  - Bayesian classification
  - Support vector machines
  - Decision trees
  - Neural networks
  - k-nearest neighbor

**Which one to use?**

### Weak and strong classifier?






## Boosting

- Can an ensemble of weak learners get a strong one

$$H(x) = sign(h^1(x) + h^2(x) + h^3(x) + ...)$$

- What is error rate?

$$\epsilon = \frac{1}{m}\sum_{i=1}^{m} 1_{H(x^{(i)}) \neq y^{(i)}} = \sum_{wrong} \frac{1}{m}$$

Three weak learners, having error on different data points is prefect

- But, most likely it is not going to be the case
- We can do the following
  1. Get weak learner $h^1$ (that is best)
  2. Exaggerate the data where $h^1$ errs and get weak learner $h^2$
  3. Exaggerate again the data for $h^1 \neq h^2$ and get weak learner $h^3$
- Will this work?



## Boosting

- Boosting uses ensemble of learners (weak ones)
- AdaBoost[1] is one of the widely used boosting algorithm
- Given with $m$ labeled training examples $(x^{(1)}, y^{(1)}), ..., (x^{(m)}, y^{(m)})$ where the $x^{(i)} \in \mathcal{X}$, and the labels $y^{(i)} \in \{-1, +1\}$
- Round $t$ computes a distribution $D_t$ over training examples. Weak learning algorithm is applied to find a suitable (low weighted error $\epsilon_t$ relative to $D_t$) weak hypothesis $h_t : \mathcal{X} \to \{-1, +1\}$
- Final or combined hypothesis $H$ computes the sign of a weighted combination of weak hypotheses

$$H(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

[1] A decision-theoretic generalization of on-line learning and an application to boosting, Freund, Yoav and Schapire, Robert E, European conference on computational learning theory, pp 23–37 Springer(1995)



## Boosting

- Let each data point have a weight $w_i$ associated with them. Initially $w_i = \frac{1}{m}$ so that error rate becomes

$$\epsilon = \sum_{wrong} \frac{1}{m} = \sum_{i \in wrong} w_i$$

- Sum of weights is 1

$$\sum w_i = 1$$

- Weights are modified in every round



## AdaBoost Algorithm

**Algorithm 1:** AdaBoost

1. Initialize $w_1, ..., w_m$ to $\frac{1}{m}$
2. **for** *round* $t = 1$ *to* $T$ **do**
3. Choose hypothesis $h^t$ minimizing error $\epsilon^t$ on current distribution
4. Compute $\alpha_t = \frac{1}{2} \ln\left(\frac{1-\epsilon^t}{\epsilon^t}\right)$
5. Update all $w_1, ..., w_m$ values
6. **return** $h$ and $\alpha$

- Update of weights $w_i$ at time $t$ is done using

$$w_i^{t+1} = \frac{w_i^t}{z_t} e^{-(\alpha_t h^t(x^{(i)}) y^{(i)})}$$

for all data point *i.e.* value of $i$ varies from 1 to $m$

- Here $z_t$ is a normalization factor

## Normalization factor

- Here $z_t$ is a normalization factor so

$$\begin{aligned} z_t &= \sum_{i=1}^{m} w_i^t e^{-(\alpha_t h^t(x^{(i)}) y^{(i)})} \\ &= \sum_{i \in right} w_i^t e^{-(\alpha_t h^t(x^{(i)}) y^{(i)})} + \sum_{i \in wrong} w_i^t e^{-(\alpha_t h^t(x^{(i)}) y^{(i)})} \\ &= \sum_{i \in right} w_i^t e^{-(\alpha_t)} + \sum_{i \in wrong} w_i^t e^{-(\alpha_t(-1))} \\ &= \sum_{i \in right} w_i^t \sqrt{\frac{\epsilon^t}{1-\epsilon^t}} + \sum_{i \in wrong} w_i^t \sqrt{\frac{1-\epsilon^t}{\epsilon^t}} \\ &= \sqrt{\frac{\epsilon^t}{1-\epsilon^t}} \sum_{i \in right} w_i^t + \sqrt{\frac{1-\epsilon^t}{\epsilon^t}} \sum_{i \in wrong} w_i^t \\ &= \sqrt{\frac{\epsilon^t}{1-\epsilon^t}}(1-\epsilon^t) + \sqrt{\frac{1-\epsilon^t}{\epsilon^t}}\epsilon^t \\ &= 2\sqrt{\epsilon^t(1-\epsilon^t)} \end{aligned}$$



## Boosting

- Weight update for correctly classified data point

$$w_i^{t+1} = w_i^t \frac{1}{2(1-\epsilon^t)}$$

- Weight update is therefore for wrongly classified data point

$$w_i^{t+1} = w_i^t \frac{1}{2\epsilon^t}$$

### Final hypothesis

$$H(x) = sign(\alpha_1 h^1(x) + \alpha_2 h^2(x) + \alpha_3 h^3(x) + ...)$$

$$\epsilon^t = \sum_{i \in wrong} w_i^t$$



## AdaBoost at work

Consider the following data set

| | ( $x$ | $y$) |
|---|---|---|
| $s_1$ | (1, | -1) |
| $s_2$ | (2, | -1) |
| $s_3$ | (3, | +1) |
| $s_4$ | (4, | +1) |
| $s_5$ | (5, | +1) |
| $s_6$ | (6, | +1) |
| $s_7$ | (7, | +1) |
| $s_8$ | (8, | -1) |
| $s_9$ | (9, | -1) |
| $s_{10}$ | (10, | -1) |

- **Round-00:** initialize weights

| $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$ | $w_8$ | $w_9$ | $w_{10}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |

- **Round-01:** Let sampling according to w produces (3, 10, 8, 2, 7, 5, 1, 3, 8, 3) so following sub-set of data is considered

| | | |
|---|---|---|
| $s_1$ | (1, | -1) |
| $s_2$ | (2, | -1) |
| $s_3$ | (3, | +1) |
| $s_5$ | (5, | +1) |
| $s_7$ | (7, | +1) |
| $s_8$ | (8, | -1) |
| $s_{10}$ | (10, | -1) |

Consider various hypothesis



## AdaBoost at work (Round-01)

| | | | $h_a$ | $h_b$ | $h_c$ | $h_d$ | $h_e$ | $h_f$ | $h_g$ | $h_h$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $s_1$ | (1, | -1) | +1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_2$ | (2, | -1) | +1 | +1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_3$ | (3, | +1) | +1 | +1 | +1 | -1 | -1 | -1 | -1 | -1 |
| $s_5$ | (5, | +1) | +1 | +1 | +1 | +1 | -1 | -1 | -1 | -1 |
| $s_7$ | (7, | +1) | +1 | +1 | +1 | +1 | +1 | -1 | -1 | -1 |
| $s_8$ | (8, | -1) | +1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 |
| $s_{10}$ | (10, | -1) | +1 | +1 | +1 | +1 | +1 | +1 | +1 | -1 |

- Select $h_c$ as $h1$
- What is decision threshold? 2.5
- Compute error on whole dataset



## AdaBoost at work (Round-01)

Threshold is 2.5

| | ( $x$ | $y$) | $h_1$ |
|---|---|---|---|
| $s_1$ | (1, | -1) | -1 |
| $s_2$ | (2, | -1) | -1 |
| $s_3$ | (3, | +1) | +1 |
| $s_4$ | (4, | +1) | +1 |
| $s_5$ | (5, | +1) | +1 |
| $s_6$ | (6, | +1) | +1 |
| $s_7$ | (7, | +1) | +1 |
| $s_8$ | (8, | -1) | +1 |
| $s_9$ | (9, | -1) | +1 |
| $s_{10}$ | (10, | -1) | +1 |

- Error rate $\epsilon = \sum_{i \in wrong} w_i$
  $= w_8 + w_9 + w_{10} = 0.1 + 0.1 + 0.1 = 0.3$
- $\alpha = \frac{1}{2} \ln\left(\frac{1-\epsilon}{\epsilon}\right) = \frac{1}{2} \ln\left(\frac{1-0.3}{0.3}\right) = 0.4236$
- Weights are modified according to

$$w_i = w_i \times \begin{cases} \frac{1}{2(1-\epsilon)} = 0.7142 & \text{correct} \\ \frac{1}{2\epsilon} = 1.6666 & \text{wrong} \end{cases}$$

| $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$ | $w_8$ | $w_9$ | $w_{10}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.17 | 0.17 | 0.17 |



## AdaBoost at work (Round-02)

Consider the data set

| | ( $x$ | $y$) |
|---|---|---|
| $s_1$ | (1, | -1) |
| $s_2$ | (2, | -1) |
| $s_3$ | (3, | +1) |
| $s_4$ | (4, | +1) |
| $s_5$ | (5, | +1) |
| $s_6$ | (6, | +1) |
| $s_7$ | (7, | +1) |
| $s_8$ | (8, | -1) |
| $s_9$ | (9, | -1) |
| $s_{10}$ | (10, | -1) |

- **Round-02:** Let sampling according to new w produces (8, 10, 7, 10, 3, 1, 10, 8, 4, 9) so following sub-set of data is considered

| | | |
|---|---|---|
| $s_1$ | (1, | -1) |
| $s_3$ | (3, | +1) |
| $s_4$ | (4, | +1) |
| $s_7$ | (7, | +1) |
| $s_8$ | (8, | -1) |
| $s_9$ | (9, | -1) |
| $s_{10}$ | (10, | -1) |

Consider various hypothesis

## AdaBoost at work (Round-02)

| | | | $h_a$ | $h_b$ | $h_c$ | $h_d$ | $h_e$ | $h_f$ | $h_g$ | $h_h$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $s_1$ | (1, | -1) | +1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_3$ | (3, | +1) | +1 | +1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_4$ | (4, | +1) | +1 | +1 | +1 | -1 | -1 | -1 | -1 | -1 |
| $s_7$ | (7, | +1) | +1 | +1 | +1 | +1 | -1 | -1 | -1 | -1 |
| $s_8$ | (8, | -1) | +1 | +1 | +1 | +1 | +1 | -1 | -1 | -1 |
| $s_9$ | (9, | -1) | +1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 |
| $s_{10}$ | (10, | -1) | +1 | +1 | +1 | +1 | +1 | +1 | +1 | -1 |

- Select $h_h$ as $h2$
- What is decision threshold? 10.5
- Compute error on whole dataset



## AdaBoost at work (Round-02)

Threshold is 10.5

| | ( $x$ | $y$) | $h_2$ |
|---|---|---|---|
| $s_1$ | (1, | -1) | -1 |
| $s_2$ | (2, | -1) | -1 |
| $s_3$ | (3, | +1) | -1 |
| $s_4$ | (4, | +1) | -1 |
| $s_5$ | (5, | +1) | -1 |
| $s_6$ | (6, | +1) | -1 |
| $s_7$ | (7, | +1) | -1 |
| $s_8$ | (8, | -1) | -1 |
| $s_9$ | (9, | -1) | -1 |
| $s_{10}$ | (10, | -1) | -1 |

- Error rate $\epsilon = \sum_{i \in wrong} w_i$
  $= w_3 + w_4 + w_5 + w_6 + w_7 = 5 \times 0.07 = 0.35$
- $\alpha = \frac{1}{2} \ln\left(\frac{1-\epsilon}{\epsilon}\right) = \frac{1}{2} \ln\left(\frac{1-0.35}{0.35}\right) = 0.3095$
- Weights are modified according to

$$w_i = w_i \times \begin{cases} \frac{1}{2(1-\epsilon)} = 0.7692 & \text{correct} \\ \frac{1}{2\epsilon} = 1.4285 & \text{wrong} \end{cases}$$

| $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$ | $w_8$ | $w_9$ | $w_{10}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.17 | 0.17 | 0.17 |
| 0.037 | 0.037 | 0.177 | 0.177 | 0.177 | 0.177 | 0.177 | 0.091 | 0.091 | 0.091 |



## AdaBoost at work (Round-03)

Consider the data set

| | ( $x$ | $y$) |
|---|---|---|
| $s_1$ | (1, | -1) |
| $s_2$ | (2, | -1) |
| $s_3$ | (3, | +1) |
| $s_4$ | (4, | +1) |
| $s_5$ | (5, | +1) |
| $s_6$ | (6, | +1) |
| $s_7$ | (7, | +1) |
| $s_8$ | (8, | -1) |
| $s_9$ | (9, | -1) |
| $s_{10}$ | (10, | -1) |

- **Round-03:** Let sampling according to new w produces (8, 7, 4, 8, 6, 9, 5, 8, 3, 4) so following sub-set of data is considered

| | ( $x$ | $y$) |
|---|---|---|
| $s_3$ | (3, | +1) |
| $s_4$ | (4, | +1) |
| $s_5$ | (5, | +1) |
| $s_6$ | (6, | +1) |
| $s_7$ | (7, | +1) |
| $s_8$ | (8, | -1) |
| $s_9$ | (9, | -1) |



## AdaBoost at work (Round-03)

| | | | $h_a$ | $h_b$ | $h_c$ | $h_d$ | $h_e$ | $h_f$ | $h_g$ | $h_h$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $s_3$ | (3, | +1) | +1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_4$ | (4, | +1) | +1 | +1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_5$ | (5, | +1) | +1 | +1 | +1 | -1 | -1 | -1 | -1 | -1 |
| $s_6$ | (6, | +1) | +1 | +1 | +1 | +1 | -1 | -1 | -1 | -1 |
| $s_7$ | (7, | +1) | +1 | +1 | +1 | +1 | +1 | -1 | -1 | -1 |
| $s_8$ | (8, | -1) | +1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 |
| $s_9$ | (9, | -1) | +1 | +1 | +1 | +1 | +1 | +1 | +1 | -1 |

- Select $h_a$ as $h3$
- What is decision threshold? 0.5
- Compute error on whole dataset



## AdaBoost at work (Round-03)

Threshold is 0.5

| | ( $x$ | $y$) | $h_3$ |
|---|---|---|---|
| $s_1$ | (1, | -1) | +1 |
| $s_2$ | (2, | -1) | +1 |
| $s_3$ | (3, | +1) | +1 |
| $s_4$ | (4, | +1) | +1 |
| $s_5$ | (5, | +1) | +1 |
| $s_6$ | (6, | +1) | +1 |
| $s_7$ | (7, | +1) | +1 |
| $s_8$ | (8, | -1) | +1 |
| $s_9$ | (9, | -1) | +1 |
| $s_{10}$ | (10, | -1) | +1 |

- Error rate $\epsilon = \sum_{i \in wrong} w_i$
  $= 2 \times 0.037 + 3 \times 0.091 = 0.34$
- $\alpha = \frac{1}{2} \ln\left(\frac{1-\epsilon}{\epsilon}\right) = \frac{1}{2} \ln\left(\frac{1-0.34}{0.34}\right) = 0.3316$
- We may continue for next round like that
- Let use see our accuracy now



## AdaBoost at work (Round-03)

$$(\alpha_1, \alpha_2, \alpha_3) = (0.4236, 0.3095, 0.3316)$$

| | ( $x$ | $y$) | $h_1$ | $h_2$ | $h_3$ | $sign(\sum_i(\alpha_i \times h_i))$ |
|---|---|---|---|---|---|---|
| $s_1$ | (1, | -1) | -1 | -1 | +1 | -1 |
| $s_2$ | (2, | -1) | -1 | -1 | +1 | -1 |
| $s_3$ | (3, | +1) | +1 | -1 | +1 | +1 |
| $s_4$ | (4, | +1) | +1 | -1 | +1 | +1 |
| $s_5$ | (5, | +1) | +1 | -1 | +1 | +1 |
| $s_6$ | (6, | +1) | +1 | -1 | +1 | +1 |
| $s_7$ | (7, | +1) | +1 | -1 | +1 | +1 |
| $s_8$ | (8, | -1) | +1 | -1 | +1 | +1 |
| $s_9$ | (9, | -1) | +1 | -1 | +1 | +1 |
| $s_{10}$ | (10, | -1) | +1 | -1 | +1 | +1 |

## AdaBoost Summary

**Algorithm 2:** AdaBoost

1 Initialize $w_1, ..., w_m$ to $\frac{1}{m}$
2 **for** *round* $t = 1$ *to* $T$ **do**
3 Choose hypothesis $h^t$ minimizing error $\epsilon^t$ on current distribution
4 Compute $\alpha_t = \frac{1}{2} \ln \left(\frac{1-\epsilon^t}{\epsilon^t}\right)$
5 Update all $w_1, ..., w_m$ values
6 **return** $h$ and $\alpha$

$$\epsilon^t = \sum_{i \in wrong} w_i^t$$

$$w_i^{t+1} = \frac{w_i^t}{z_t} e^{-(\alpha_t h^t(x^{(i)}) y^{(i)})}$$

$$z_t = 2\sqrt{\epsilon^t(1-\epsilon^t)}$$

$$w_i^{t+1} = \begin{cases} w_i^t \frac{1}{2(1-\epsilon^t)} & \text{correctly classified data point} \\ w_i^t \frac{1}{2\epsilon^t} & \text{otherwise} \end{cases}$$

### Final hypothesis

$$H(x) = sign(\alpha_1 h^1(x) + \alpha_2 h^2(x) + \alpha_3 h^3(x) + ...)$$



## Bagging

- A technique that repeatedly samples (with replacement) $m$ training data points according the uniform distribution
- Some data points may be repeated or omitted. Sample set is expected to have 63% of items [2]

**Algorithm 3:** Bagging

1 Let $k$ be the number of bootstrap samples
2 **for** $i = 1$ *to* $k$ **do**
3 Create $D_i$; a bootstrap sample of size $m$
4 Train classifier $C_i$ on $D_i$
5 **return** $C_i$'s

$$C(x) = \underset{y}{\text{argmax}} \sum_i \delta(C_i(x) = y) \qquad \delta(true) = 1; otherwise\ 0$$

[2] An item has the probability $1 - (1 - \frac{1}{m})^m$ of being selected that converges to $1 - \frac{1}{e}$ as $m$ increases



## Bagging Example

Consider the following data set

| | ( $x$ | $y$) |
|---|---|---|
| $s_1$ | (0.1, | +1) |
| $s_2$ | (0.2, | +1) |
| $s_3$ | (0.3, | +1) |
| $s_4$ | (0.4, | -1) |
| $s_5$ | (0.5, | -1) |
| $s_6$ | (0.6, | -1) |
| $s_7$ | (0.7, | -1) |
| $s_8$ | (0.8, | +1) |
| $s_9$ | (0.9, | +1) |
| $s_{10}$ | (1.0, | +1) |

Let sampling gets data points as $s_1$, $s_2$, $s_3$, $s_4$, $s_5$, $s_6$, $s_9$, $s_{10}$ Then

| ( $x$ $y$) | 0 | .15 | .25 | .35 | .45 | .55 | .65 | .75 | 1.05 |
|---|---|---|---|---|---|---|---|---|---|
| $s_1$ (0.1,+1) | +1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_2$ (0.2,+1) | +1 | +1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_3$ (0.3,+1) | +1 | +1 | +1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $s_4$ (0.4,-1) | +1 | +1 | +1 | +1 | -1 | -1 | -1 | -1 | -1 |
| $s_5$ (0.5,-1) | +1 | +1 | +1 | +1 | +1 | -1 | -1 | -1 | -1 |
| $s_6$ (0.6,-1) | +1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 | -1 |
| $s_9$ (0.9,+1) | +1 | +1 | +1 | +1 | +1 | +1 | +1 | -1 | -1 |
| $s_{10}$(1.0,+1) | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | -1 |

Best classifier can be obtained by taking threshold at 0.35 and reversing the sign.

$$Classification = \begin{cases} +1 & \text{if } x \leq 0.35 \\ -1 & \text{otherwise} \end{cases}$$



## Bagging Example

Similarly

| | ( $x$ | $y$) |
|---|---|---|
| $s_1$ | (0.1, | +1) |
| $s_2$ | (0.2, | +1) |
| $s_3$ | (0.3, | +1) |
| $s_4$ | (0.4, | -1) |
| $s_5$ | (0.5, | -1) |
| $s_6$ | (0.6, | -1) |
| $s_7$ | (0.7, | -1) |
| $s_8$ | (0.8, | +1) |
| $s_9$ | (0.9, | +1) |
| $s_{10}$ | (1.0, | +1) |

Bagging Round 1:

| x | 0.1 | 0.2 | 0.2 | 0.3 | 0.4 | 0.4 | 0.5 | 0.6 | 0.9 | 0.9 | x <= 0.35 ==> y = 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | x > 0.35 ==> y = -1 |

Bagging Round 2:

| x | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.8 | 0.9 | 1 | 1 | 1 | x <= 0.65 ==> y = 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | x > 0.65 ==> y = 1 |

Bagging Round 3:

| x | 0.1 | 0.2 | 0.3 | 0.4 | 0.4 | 0.5 | 0.7 | 0.7 | 0.8 | 0.9 | x <= 0.35 ==> y = 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | x > 0.35 ==> y = -1 |

Bagging Round 4:

| x | 0.1 | 0.1 | 0.2 | 0.4 | 0.4 | 0.5 | 0.5 | 0.7 | 0.8 | 0.9 | x <= 0.3 ==> y = 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | x > 0.3 ==> y = -1 |

Bagging Round 5:

| x | 0.1 | 0.1 | 0.2 | 0.5 | 0.6 | 0.6 | 0.6 | 1 | 1 | 1 | x <= 0.35 ==> y = 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | x > 0.35 ==> y = -1 |

Bagging Round 6:

| x | 0.2 | 0.4 | 0.5 | 0.6 | 0.7 | 0.7 | 0.7 | 0.8 | 0.9 | 1 | x <= 0.75 ==> y = -1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | x > 0.75 ==> y = 1 |

Bagging Round 7:

| x | 0.1 | 0.4 | 0.4 | 0.6 | 0.7 | 0.8 | 0.9 | 0.9 | 0.9 | 1 | x <= 0.75 ==> y = -1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | x > 0.75 ==> y = 1 |

Bagging Round 8:

| x | 0.1 | 0.2 | 0.5 | 0.5 | 0.5 | 0.7 | 0.7 | 0.8 | 0.9 | 1 | x <= 0.75 ==> y = -1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | x > 0.75 ==> y = 1 |

Bagging Round 9:

| x | 0.1 | 0.3 | 0.4 | 0.4 | 0.6 | 0.7 | 0.7 | 0.8 | 1 | 1 | x <= 0.75 ==> y = -1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | x > 0.75 ==> y = 1 |

Bagging Round 10:

| x | 0.1 | 0.1 | 0.1 | 0.1 | 0.3 | 0.3 | 0.8 | 0.8 | 0.9 | 0.9 | x <= 0.05 ==> y = -1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | x > 0.05 ==> y = 1 |



## Bagging Example

x <= 0.35 ==> y = 1
x > 0.35 ==> y = -1

x <= 0.65 ==> y = 1
x > 0.65 ==> y = 1

x <= 0.35 ==> y = 1
x > 0.35 ==> y = -1

x <= 0.3 ==> y = 1
x > 0.3 ==> y = -1

x <= 0.35 ==> y = 1
x > 0.35 ==> y = -1

x <= 0.75 ==> y = -1
x > 0.75 ==> y = 1

x <= 0.75 ==> y = -1
x > 0.75 ==> y = 1

x <= 0.75 ==> y = -1
x > 0.75 ==> y = 1

x <= 0.75 ==> y = -1
x > 0.75 ==> y = 1

x <= 0.05 ==> y = -1
x > 0.05 ==> y = 1

| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Round** | **x=0.1** | **x=0.2** | **x=0.3** | **x=0.4** | **x=0.5** | **x=0.6** | **x=0.7** | **x=0.8** | **x=0.9** | **x=1.0** |
| 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 4 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 5 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 6 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |
| 7 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |
| 9 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |
| 10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Sum** | 2 | 2 | 2 | -6 | -6 | -6 | -6 | 2 | 2 | 2 |
| **Sign** | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |
| **True Class** | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 |



## Thank You!

**Thank you very much for your attention!**

**Queries ?**